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SOME EMPIRICAL EVIDENCE ON TWO SECTOR GROWTH

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Introduction

One ot the more interesting developments of the neoclassical approach to growth has been its extension to a two sector economy in order to take into account the different characteristics of consumer and capital goods. Much of the literature stems from Uzawa's 1961 and 1963 papers and is summarized in Burmeister and Dobell (1970).

Little has been done to implement two sector growth models empirically, because national accounts data disaggregated into the two sectors are not available. Recently, however, Benos (1972) has developed estimates of the required data for the years 1947 - 62 in connection with a study of tax policy. It is the purpose of this paper to perform a more detailed analysis of that data in order to shed some light on the empirical significance of the two sector dichotomy and the existence and stability conditions available in the literature.

Some of the existence and stability conditions depend on the relationship between the savings rates out of labor and nonlabor income, and on the relative factor intensities of the two sectors. These will not be examined here because simple examination of the data will reveal that the consumption sector has a higher capital intensity, and that the savings rate out of nonlabor income is higher than the savings rate out ot labor income. Less obvious is the relationship between the production functions in the two sectors, and this is the focus of this study.

The Cobb-Douglas Case

Benos (1972) estimated a constant returns to scale Cobb-Douglas production function for each sector using ordinary least squares and omitting 1954 as an outlier. The capital stock data were adjusted for utilization using rates given in Creamer (1962). The equations were also estimated using the employment rate to adjust for capital utilization and the results did not differ significantly (Benos, 1972, p. 215).

The functions were re-estimated for this study and the results were:

$$Y_{ct} = \underset{(.1838)}{2.384}\, e^{\underset{(.0016)}{.0154t}}\, K_{ct}^{\underset{(.1036)}{.3165}}\, L_{ct}^{.6835} \qquad \bar{R}^2 = .9827 \quad DW = 2.13$$

$$S = .013 \qquad F = 398.1$$

$$Y_{kt} = \underset{(.2674)}{2.722} e^{\underset{(.0030)}{.0142t}} K_{kt}^{\underset{(.1791)}{.3098}} L_{kt}^{.6902} \qquad \bar{R}^2 = .9666 \quad DW = 1.72$$

$$S = .018 \qquad F = 203.3$$

The standard errors of the coefficients are written below them, but the standard errors of the intercepts should be compared to the logs of the reported values (.8686, 1.0012). The symbols are Y for gross private output and K for employed capital stock both measured in billions of 1954 dollars. Employment is measured in millions of full force equivalent employees, and technical progress is captured by a dummy variable taking values 0.1, - - -. The subscript c identifies the consumption sector; the subscript k identifies the capital goods sector.

Both functions fit the data well as measured by the adjusted coefficients of determination ($\bar{R}^2$), the standard errors of estimate and the F ratios. All coefficients are significant at the five percent level except the capital elasticity for the capital goods sector. The fit of the production function for this sector is not as good as the fit to the consumption sector. The Durbin Watson statistics indicate that serial correlation of the residuals is not a problem. The estimates of the coefficients reported here are slightly different than those reported in Benos (1972) and presumably reflects the different computers and software employed in the two studies.

The striking feature of the two estimated production functions is the similarity of the estimated coefficients for the two sectors. Expressed as percentages of the standard errors of the equation for the consumption sector, the intercepts differ by 72 percent, and the capital elasticities differ by 7 percent. The differences expressed as percentages of the standard errors of the equation for the capital sector are considerably lower in all cases.

If the two production functions are the same, the two sector growth model collapses to a one sector model. Any given wage rental ratio will lead to the same capital labor ratio in both sectors (in efficiency units) and the same output per efficiency unit with a relative price ratio of unity. Nothing will have been gained by recognizing the different characteristics of consumer and capital goods. Thus, it is important to test the hypothesis that the two functions are the same. To do this we use the Chow (1960) test which tests the significance of the reduction in the residual variance from the overall regression adhered by estimating residual regressions (Dutta, 1975, p. 174).

The results of estimating the overall regression (combining the data for the two sectors) were:

$$Y_t = \underset{(.043)}{4.917} e^{\underset{(.0008)}{.021t}} K_t^{\underset{(.026)}{-.089}} L_t^{1.089} \quad \bar{R}^2 = .9602 \quad DW = 1.97$$

$$S = .019 \quad F = 350.9$$

The residual sums of squares were .002 for the consumption, .0037 ıor the capital and .01 for the overall regressions. With 30 observations the F ratio of 6.03 has 3 and 24 degrees of freedom and the tabulated value at the 5 percent level is less than half this. Thus, we reject the hypothesis that there is no significant difference between the two production functions.

The Chow test does not identify the source of the difference between the regressions. Recent work by Gujarati (1970a and 1970b) shows the relation between this test and the use of dummy variables. It was therefore decided to re-estimate the overall regression incorporating dummy variables for shifts in the coefficients. The results were:

$$\ln Y_t/L_t = \underset{(.2352)}{1.001} \; \underset{(.3211)}{-.1328} \, Z_{At} + \underset{(.1575)}{.3098} \ln K_t/L_t + \underset{(.2000)}{.0068} \, Z_{Kt}$$
$$+ \underset{(.0026)}{.0142t} \; \underset{(.0038)}{-.0012 Z_t}$$
$$\bar{R}^2 = .9745 \quad DW = 2.00 \quad S = .015 \quad F = 223.0$$

The equation is presented in log form to permit comparison of the standard errors to their coefficients. The estimated intercept and coefficients of $\ln K_t/L_t$ and t are the same as those reported for the production function oı the capital sector. The dummy variables $Z_{A'}$, Z_K and Z_t allow for shifts in the intercept, the capital elasticity and the rate of technical progress when the data refer to the consumption sector. None of the t ratios for the shift variables is significant at the 5 percent level. This does not contradict the Chow test, of course, because the latter is based on a simultaneous confidence interval, not the intersection of individual intervals.

While it is possible that all coefficients are truly different for the two sectors, the failure of any of the dummy variables to be significant in the above equation suggests that one or more of the coefficients may be the same. To examine this six more regressions were estimated using different subsets of the three dummy variables. The results are presented in Table 1.

TABLE 1

Selected Cobb Douglas Production Functions estimated using dummy variables

Intercept	Z_A	l_nK/L	Z_K	t	Z_t	$\bar{R}^2$	S	F	DW
1.0658 (.1493)	—.2378 (.1309)	.2661 (.0987)	.0736 (.0726)	.0150 (.0015)	ni [a]	.9754	.015	288.8	2.00
.9950 (.1421)	—.1219 (.0287)	.3140 (.0951)	ni [a]	.0141 (.0017)	.0013 (.0012)	.9756	.015	290.3	2.01
.9301 (.1574)	ni [a]	.3574 (.1055)	—.0755 (.0179)	.0134 (.0019)	.0024 (.0013)	.9754	.015	288.3	2.01
.0162 (.1411)	—.1077 (.0256)	.2967 (.0940)	ni [a]	.0150 (.0015)	ni [a]	.9754	.015	384.3	1.94
1.0272 (.1542)	ni [a]	.2872 (.1022)	—.0558 (.0148)	.0155 (.0015)	ni [a]	.9733	.016	352.8	1.86
1.5490 (.0722)	ni [a]	.0626 (.0440)	ni [a]	.0209 (.0009)	—.0010 (.0013)	.9596	.019	230.3	1.92

[a] ni = not included in the regression

The first row of Table 1 reports the results of estimating the equation with dummy variables which shift the intercept and the capital elasticity across sectors, but require the rates of neutral technical progress to be the same. The fit of this equation is slighter better than the fit reported above, and the common rate of technical progress (5.1 percent per annum) is an average of the two previously reported rates. The remaining shift variables are still not statistically significant although their t ratios are considerably higher.

The spreads between the intercepts and the capital elasticities for the two sectors greatly increased between the two regressions. This is particularly true of the capital elasticity and reflects a sharp reduction in the value for the capital sector. The result is a capital elasticity for consumption of .34 compared to .27 for the capital sector. The higher value for the consumption sector implies that, for any given wage rental ratio, the capital labor ratio in the consumption sector will be larger than the corresponding ratio in the capital sector. In this sense, the consumption sector will appear to be more capital intensive. This is sometimes given as the stability condition for a two sector growth model with fixed coefficients production functions (Shinkai 1960, Hicks 1965). The condition is not required for stability when Cobb-Douglas functions are used, however.

The elimination of the dummy variable for the rate of technical progress reduces the complexity of the model significantly. The technological progress incorporated in the equations reported here is of the disembodied type; in the words of Allen (1968) it "falls like manna from heaven" coating existing factors of production in a way that makes them more productive. Equal rates of technical progress imply that the manna falls at the same rate on firms producing consumer goods as it has on firms producing capital goods. If disembodied technical progress is identified with better management techniques, the equal rates of progress imply that the new techniques are applied at the same rate in both sectors. It is interesting that a recent textbook exposition of the two sector model with technical progress (Burmeister and Dobell 1970) makes the assumption of a common rate of technical progress for the purpose of simplicity. The data suggest that the assumption may be reasonable as well as simplifying.

Most of the other regressions reported in Table 1 are less satisfactory. The third and fifth equations retain the dummy for the capital elasticity (with and without the dummy for the rate of technical progress). Both of these equations yield capital elasticities that are smaller for the consumption than the capital sectors. Thus, given any wage rental ratio the capital labor ratio in the consumption sector (in efficiency units) will be lower than in the capital sector. While this does not imply instability, it does contradict the expected result. The last equation has a negative elasticity and a poorer fit than the others. The fourth equation retains the intercept dummy only and is therefore trivial. The second equation retains different rates of technical progress but assumes a common capital elasticity. This is difficult to rationalize.

In conclusion, we have shown that the estimated Cobb-Douglas production function for the consumption sector is significantly different from that estimated for the capital sector. None of the estimated coefficients for either sector is as much as one standard error different from its corresponding value for the other sector. If we were to assume that at least one of the coefficients was equal for both sectors, the most reasonable assumption would be a common rate of disembodied technical progress for both sectors.

CES Production Function Case

Data on capital and labor income for each sector are also presented by Benos (1972). If profit maximization and perfect competition are assumed for each sector (and these assumptions are made as part of the model in any case) these data may be used to estimate the parameters of CES production functions for both sectors (Walters 1968).

Once the assumption of a Cobb-Douglas specification for the production functions is relaxed, the type of disembodied technical progress incorporated in the model determines the statistical specification of the production function. Specifically, the technical progress may be Harrod, Solow or Hicks neutral. A production function with disembodied technical progress contains three arguments: capital (K), labor (L) and technology (t). If the technical progress is Harrod neutral then it appears to augment the labor input. That is, each laborer grows more productive over time. This is usually captured by writing the labor input as $\bar{L}$ where $\bar{L} = e^{\lambda t}L$. The coefficient of L grows over time at a constant rate, and thus the labor input would grow even if the labor force remained constant. The productivity of laborers already on the job rises as a result of the technological progress.

If the technological progress is Solow neutral then it appears to augment the capital input. The interpretation is the same as that given above for the Harrod neutral case, with the word labor replaced by capital. The capital input can be written $\overline{K} = e^{\lambda t}K$. If the technological progress is Hicks neutral then it appears to augment both the labor and capital inputs, but at equal rates. Thus the arguments of the production function are $\overline{K} = e^{\lambda t}K$ and $\overline{L} = e^{\lambda t}L$.

Because a Cobb-Douglas production function is multiplicative it can be written $Ae^{\lambda^* t}K^{\alpha}L^{1-\alpha}$ regardless of whether the technical progress is Harrod, Solow or Hicks neutral. In the Harrod neutral case we have

$$AK^{\alpha}\overline{L}^{1-\alpha} = AK^{\alpha}(e^{\lambda t}L)^{1-\alpha} = AK^{\alpha}\, e^{\lambda(1-\alpha)t}L^{1-\alpha} = Ae^{\lambda(1-\alpha)t}K^{\alpha}L^{1-\alpha}$$

so that $\lambda^* = \lambda(1\text{-}\alpha)$. Since estimates of α and λ^* are yielded by the regression, an estimate of λ is easily obtained. In the case of Solow neutral progress $\lambda^* = \lambda/\alpha$ and again an estimate of λ is easily determined. In the Hicks neutral case, $\lambda^* = \lambda$.

One implication ot the irrelevance of the specification of neutral technical change for the statistical specification of the Cobb-Douglas production function is that we cannot use the latter function to decide which specification of progress to use. The problem is simplified statistically but at the cost of failing to suggest the appropriate specification of technology.

The CES production function is additive, not multiplicative, so the production functions with Harrod, Hicks and Solow neutral specifications are not observationally equivalent. The three equations are (excluding the disturbances):

$$\text{Harrod:}\quad Y_t = \gamma\,\{\delta K_t^{-\rho} + (1\text{-}\delta)(e^{\lambda t}L_t)^{-\rho}\}^{-\frac{1}{\rho}}$$

$$\text{Solow:}\quad Y_t = \gamma\,\{\delta(e^{\lambda t}K_t)^{-\rho} + (1\text{-}\delta)\,L_t^{-\rho}\}^{-\frac{1}{\rho}}$$

$$\text{Hicks:}\quad Y_t = \gamma e^{\lambda t}\{\delta K_t^{-\rho} + (1\text{-}\delta)L_t^{-\rho}\}^{-\frac{1}{\rho}}$$

All three equations are not estimated here. We confine our attention to the Harrod neutral and Hicks neutral cases, since these are the ones discussed in the literature (Diamond 1965, Takayama 1965). Hicks neutral technical change was the first one introduced into the general economics literature, and it is still used extensively even in growth theory. Harrod neutral technical progress was the first type incorporated into the growth theory literature (Harrod 1939) and it is necessary for the existence of a steady state in a one sector neoclassical model. Solow neutral progress is only encountered in the literature on embodied technical change.

The results for Harrod neutral progress are discussed first. The marginal product of capital is $(\delta Y_t K_t^{-\rho-1}) / \{\delta K_t^{-\rho} + (1\text{-}\delta)\ (e^{\gamma t}L_t)^{-\rho}\}$ and the marginal product of labor is $[(1\text{-}\delta)\ Y_t L_t^{-\rho-1} e^{-\lambda\rho t}] / \{\delta K_t^{-\rho} + (1\text{-}\delta)$ $(e^{\lambda t}L_t)^{-\rho}\}$. Hence the marginal rate of substitution (ignoring the sign) is

$$\frac{\partial Y/\partial K}{\partial Y/\partial L} = \frac{\delta}{1\text{-}\delta}\left(\frac{K_t}{L_t}\right)^{-\rho}\left(\frac{L_t}{K_t}\right)^{e^{\lambda\rho t}}$$

Equating this to the inverse of the wage rental ratio and multiplying both sides by K_t/L_t yields

$$\frac{rK_t}{wL_t} = \frac{\delta}{1\text{-}\delta}\left(\frac{K_t}{L_t}\right)^{-\rho e^{\lambda\rho t}}$$

The left hand side is capital income divided by labor income (in a profit maximizing perfectly competive industry). Taking logs of both sides yields an estimable equation.

Estimating the above equation for the two sectors yielded

$$\ln\frac{rK_c}{wL_c} = \underset{(.5685)}{-1.8577} + \underset{(.3203)}{.5598}\ \ln\frac{K_c}{L_c}\ \underset{(.0049)}{-.022t} \qquad \bar{R}^2 = .7594\ \ DW{=}2.00 \quad S = .04\ \ F = 23.1$$

$$\ln\frac{rK_k}{wL_k} = \underset{(.8121)}{-2.2207} + \underset{(.5440)}{.6396}\ \ln\frac{K_k}{L_k}\ \underset{(.0091)}{-.0316t} \qquad \bar{R}^2 = .7952\ \ DW{=}2.17 \quad S{=}.05\ \ F = 28.2$$

The fits of these equations are considerably lower than those found for the Cobb-Douglas case. Since the independent variables are the same for the CES and Cobb-Douglas cases, this reduction in fit reflects the greater variance of the ratio of factor shares relative to output per head. The capital labor ratio coefficient is considerably less than twice its standard error for both equations, but the coefficients of technological progress are significant in the two sectors.

The estimated coefficients in the two sectors appear less close for the CES functions than they did for the Cobb-Douglas, but when allowance is made for the larger standard errors the two estimated functions again appear very similar. Expressed as percentages of the standard errors in the capital sector, the intercept difference is 45 percent, the difference between the coefficients of the capital labor ratio is about 15 percent, and the difference between the coefficients of technology are 105 percent. None of these differences, therefore, is statistically significant on the basis of individual t tests.

The Chow test indicates that the two functions are significantly different. The estimated residual sum of squares are: .1027 for the combined regression, .0193 for consumption and .0340 for the capital sector. The computed F ratio is 7.5 which is over twice the tabulated value. Thus the hypothesis that the two production functions are the same is not sustained. The results of estimating the combined function with the three dummy variables defined beiore are

$$\ln \frac{rK}{wL} = \underset{(.7188)}{-2.2208} + \underset{(.9815)}{.3631Z_A} + \underset{(.4815)}{.6396} \ln K/L \underset{(.6112)}{-.798Z_K} \underset{(.0081)}{-.0316t}$$

$$+ \underset{(.0099)}{.0096Zt} \quad \bar{R}^2 = .9650 \quad DW = 2.32 \quad S = .05 \quad F = 161.2$$

This confirms the results of comparing the differences of the coefficients to their standard errors since none of the dummies is statistically significant. What is remarkable the considerable improvement in overall fit achieved by the combined equation relative to the individual ones.

Table 2 presents the results of estimating the equations with different subsets of dummy variables. On the basis of goodness of fit statistics the differences between the equations are only marginal. The coefficients of technological progress are reasonably stable for the different regressions, but the coefficient of the capital labor ratio varies dramatically.

The coefficient of the capital labor ratio is an estimate of $-\rho$, the substitution parameter in the CES production function. The fact that the estimates are all positive means that the estimates of ρ implied by the regressions are all negative. Because the elasticity of substitution is $\frac{1}{1+\rho}$ and because the estimated ρ is larger than -1 (except in the final regression) the results given here yield estimates of the elasticity of substitution which are greater than unity.

TABLE 2

Selected CES Production Functions with Harrod Neutral Technical change

Intercept	Z_A	$l_n K/L$	Z_k	t	Z_t	$\bar{R}^2$	S	F	DW
—1.6919 (.4640)	—.4983 (.4067)	.2819 (.3067)	.4680 (.2257)	—.0253 (.0047)	ni [a]	.9651	.047	201.8	2.34
—2.1469 (.4344)	.2355 (.0878)	.5901 (.2907)	ni [a]	—.0308 (.0053)	.0084 (.0036)	.9664	.046	209.7	2.32
—2.0261 (.4809)	ni [a]	.5093 (.3223)	.1454 (.0548)	—.0295 (.0058)	.0062 (.0041)	.9663	.046	208.6	2.33
—2.0076 (.4653)	.3288 (.0845)	.4762 (.3100)	ni [a]	—.0249 (.0050)	ni [a]	.9607	.050	237.5	1.96
—1.7728 (.4637)	ni [a]	.3262 (.3074)	.1968	—.0241 (.0046)	ni [a]	.9645	.048	263.5	2.20
—3.2174 (.1909)	ni [a]	1.3178 (.1164)	ni [a]	—.0438 (.0023)	.0128 (.0036)	.9584	.051	223.8	1.90

[a] ni = not included in the regression

The relatively high elasticities of substitution, if representative of the true situation, suggests that the Cobb-Douglas function is not a valid representation of production conditions in a two sector economy. On the other hand, the estimates of ρ are considerably less than twice their standard errors so that the null hypotheses cannot be rejected. Since the CES function tends to the Cobb-Douglas as $\rho \to 0$, the data are not inconsistent with the Cobb Douglas model. Further, high elasticities are surprising on the basis of annual data over a short time span. It should be noted, however, that the work of Drandakis (1964) suggests that high elasticities of substitution will result in a stable two sector model.

The estimated rate of technological progress implied by the equations reported above is obtained by dividing the coefficient of technology by the estimate of ρ. Applying this to the individual regressions yields rates of Harrod-neutral technical progress of about 4 percent per annum for the consumption sector, and about 5 percent per annum for the capital sector. These rates are considerably higher than those found for the Cobb-Douglas functions, even when the latter are converted to Harrod neutral estimates (about 2.5 percent in each sector). Insofar as the equilibrium rate of growth is often measured as the sum of the rate of technical progress plus the growth of labor, these CES estimates appear high.

The distribution parameter δ can also be calculated from the above regressions. The antilog of the intercept is $\delta/1-\delta$ which can be solved for δ. On doing this, a range of estimates of δ between .8 and .9 is obtained. As the CES function tends to the Cobb Douglas, δ tends to the capital elasticity in the latter function, which is also an estimate of capital's share (in the absence of technical progress). By this comparison, the estimates of δ found here appear high.

To summarize the comparison of the CES/Harrod neutral production functions to the Cobb-Douglas ones, the CES results yield considerably higher estimates of the elasticity of substitution, rate of technical progress and distribution parameter. There appear to be some grounds for questioning the high values found in the CES case: the poorer fit of the equations means that the Cobb-Douglas case cannot be rejected; the high elasticities are surprising on short run data; the rates of technical progress imply higher rates of growth than usually observed; and the high values of δ imply extraordinarily high capital shares.

Hicks Neutral Technical Progress

An alternative specification of technology is Hicks neutral technical progress. If this is incorporated in the production function, the marginal rate of substitution emerges as a function of the capital labor ratio alone, since the term $\gamma e^{\lambda t}$ cancels out in both the marginal products of labor and capital. The results of estimating the relative factor shares function for both sectors were (in log linear form):

$$\ln \frac{rK_c}{wL_c} = \underset{(.4170)}{.3877} \; \underset{(.2219)}{-.7243} \ln \frac{K_c}{L_c} \qquad \bar{R}^2 = .4082 \quad DW = 1.61 \quad S = .06 \quad F = 10.7$$

$$\ln \frac{rK_k}{wL_k} = \underset{(.3786)}{.4244} \; \underset{(.2347)}{-1.1493} \ln \frac{K_c}{L_c} \qquad \bar{R}^2 = .6215 \quad DW = 2.05 \quad S = 0.07 \quad F = 24.0$$

It is noteworthy that the $\bar{R}^2$, S and F goodness of fit statistics are reduced considerably when compared to the regressions including technology.

On the other hand, these regressions do not appear to be subject to the same criticisms as the Harrod neutral case. The elasticities of substitution and distribution parameters are considerably lower. The results suggest an elasticity of substitution of .58 in the consumption sector and .54 in the capital sector. Both elasticities are significantly less than unity (we reject the null hypotheses on ρ) although their sum is close to unity. These estimates are more in accord with expectations. The estimates of the distribution parameter are about .6 in both sectors, still high, but not outside the range of expectation.

The two estimated functions do not appear to be as close to each other as they were in the Cobb-Douglas or CES/Harrod-neutral cases. The Chow test, moreover, confirms that the functions are significantly different. The results of estimating the equations with the dummy variables are presented in Table 3. None of the dummy variables is significant if both are included; if one is excluded, the other is significant. Excluding the dummy variable for the intercept has little impact on the equation, whereas excluding the dummy variable for the slope has a considerable effect on the coefficients. Given the extremely low tratio for the intercept dummy (when both dummies are included) it seems likely that the source of the difference between the two equations is the difference between the substitution parameters. Thus, insofar as the two production functions are different, this reflects different elasticities of substitution.

TABLE 3

Selected CES Production Function Functions with Hicks Neutral Technical Change

Intercept	Z_A	$l_n K/L$	Z_k	$\bar{R}^2$	S	DW	F
.4244	—.0367	—1.1493	.4249	.9277	.07	1.89	125.0
(.3546)	(.5727)	(.2198)	(.3249)				
.1111	.7095	—.9549	ni[a]	.9258	.07	1.80	181.9
(.2649)	(.0503)	(.1640)					
.4104	ni[a]	—1.1406	.4042	.9304	.07	1.89	194.7
(.2732)		(.1695)	(.0277)				

[a] ni= not included in the regression

The rates of technical progress can also be obtained for the Hicks-neutral case. A "technical progress" series can be computed as $\gamma(t) = Y_{(t)} / \{ \delta K^{-\rho} + (1-\delta) L^{-\rho} \}^{-\frac{1}{\rho}}$ using the estimates of ρ and δ obtained previously. The technical progress series from the three equations reported in Table 3 were regressed against an exponential trend with dummies for the intercept (efficiency parameter) and rate of Hicks neutral technical progress.

The results are presented in Table 4. They are remarkably uniform, regardless of whether the equations were estimated using technical progress series calculated from the first, second or third equations in Table 3. They indicate that the rates of technical progress (coefficients of t) are the same in both sectors (.016). The efficiency parameters are significantly different (antilogs of the intercepts) 2.39 in the consumption sector, and 1.92 in the capital sector. It is remarkable that the CES and Cobb-Douglas production functions with Hicks neutral technical progress should yield results that suggest a common rate of progress in both sectors. Both also yield capital elasticities or distribution parameters that are similar. The essential difference between the two specifications is the lower elasticities of substitution in the CES case.

Conclusion

Cobb-Douglas and CES production functions were fit to annual output capital and employment data disaggregated into consumption and capital for the U. S. from 1947 to 1962. The Cobb-Douglas functions fit the data relatively better than the CES functions. Although the estimated coefficients were very close, the functions for the two sectors were significantly different. The results for the Cobb-Douglas and CES /Hicks neutral cases were very similar except for lower elasticities of substitution with the CES function. Both suggested very similar rates of technical progress and capital elasticities (distribution parameters) in both sectors. The major difference was in the efficiency parameter. The results suggest that the model is stable. The CES /Harrod neutral functions gave considerably higher estimates of the elasticity of substitution, distribution parameter and rate of technical progress in both sectors.

TABLE 4

Technical progress in Selected CES/Hicks-Neutral Production Functions

Series	$l_n\gamma o$	$Z\gamma o$	t	Z_t	$\bar{R}^2$	*DW*	*S*	*F*
γ_1	.8671 (.0072)	—.2155 (.0102)	.016 (.0008)	—.001 (.0011)	.9879	2.09	.02	791.5
	.8675 (.0057)	—.2163 (.0053)	.015 (.0006)	ni[a]	.9884	2.09	.02	1232.5
	.7594 (.0213)	ni[a]	.0263 (.0027)	—.0206 (.0026)	.7871	.15	.06	54.6
γ_2	.9265 (.0072)	—.3727 (.0102)	.016 (.0008)	—.0001 (.0011)	.9948	2.08	.02	1838.1
	.9270 (.0057)	—.3736 (.0053)	.016 (.0053)	ni[a]	.9950	2.08	.02	2862.2
γ_3	.8698 (.0072)	—.2234 (.0102)	.016 (.0008)	—.0001 (.0011)	.9885	2.09	.02	830.6
	.8702 (.0057)	—.2242 (.0053)	.016 (.0006)	ni[a]	.9889	2.09	.02	1293.5
	.7581 (.0221)	ni[a]	.0266 (.0028)	—.0218 (.0027)	.7828	.14	.07	53.3

γ_1 = Technological progress series obtained from the first regression in Table 4
γ_2 = Technological progress series obtained from the second regression in Table 4
γ_3 = Technological progress series obtained from the third regression in Table 4
[a] ni = not included in the regression

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