ON THE ALLOCATION OF RESOURCES FOR THE FUTURE

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Disclaimer

This is a philosophical discussion which offers structural mathematical models as a possible approach for better understanding the problems which face us. No assertion is intended here, implicit or explicit, that marketing methods and models which can be applied successfully to the sales of toothpaste or automobiles provide a definitive understanding of macro-economics. Space does not permit us to convince the reader here of the wide applicability of graphical structural models to economics and other social sciences. In particular, there are potential applications to the Greek economy.

The state of the world

The cities of the world are suffering under ever — increasing smog pollution. The population of the world is growing in an explosively exponential curve. The amount of oxygen in the air is gradually decreasing. The amount of garbage per person is rapidly increasing. The narcotics industry is one of the largest in the world. Acid rain arising from American factories is falling on Canada. The oceans, lakes and rivers of the world are becoming polluted. The intelligence level of commercial television programming continues to decrease from its present low level. The noise pollution in major cities threatens to rival the loudest of rock-and-roll.

Education

In education it is similar. School standards at all levels are woefully

shallow, and in fact there are university students who go on strike demanding still weaker academic standards. Elementary school teachers who fear and dislike mathematics are teaching the next generation of children. Gifted children are not identified and placed in challenging classes and most of them inevitably fall to the general level of mediocrity. Too many university students believe that cheating on examinations is an allowable procedure as long as one is not caught.

Politics

In politics it is also similar. Promises are made before elections and are discarded immediately after them in all countries with democratic elections. Taxation continues to be shifted in the largest countries from the rich to the poor. For example, sales, taxes, value — added taxes, and head taxes are immposed to replace the governmental income lost by "taxation reform". Governments are not run like corporations which cannot have continuous deficits. Supply-side economics have so enriched the largest corporations that they have been actively buying each other (and their own shares as well) instead of modernizing and automating their facilities to be more competitive in world markets, which the politicians predicted. Thus resources have been "trickling" up instead of down.

Graphs

All of these features are inter-related. Relationships among any collection of things can be modelled by graphs, digraphs, signed graphs and networks. This is my field of study.

Clearly our resources must be re-allocated rationally. But alas political processes are not necessarily always rational. The best that scholars can do is to analyze the global situation logically and propose algorithms for solutions to our critical problems.

My candidates for the two greatest Americal writers are Benjamin Franklin who composed proverbs such as "Three moves equal one fire" and Abraham Lincoln who replied to an invitation to speak with this:

"If you want me to talk for two hours, I can start right now. If you have one hour in mind, I'll need a day to prepare. For a 20 minute speech I'll need a week and if you want a 5 minute talk, I must have a month".

My own "Law" which has been empirically "proved" is this:

"No matter how long you think it will take, it actually takes at least three times as long".

Newspaper quotations

Even since my arrival in Greece a few days before this celebration (so that I should not fall asleep while speaking), I found three related stories in newspapers:

Athens News: China vowed to grow enough grain to feed its rising population (1,070,000,000) though arable land is diminishing and grain imports are approaching record levels.

International Herald-Tribune. Huge tracts of Africa are dying as forests get pushed back and wildlife species are driven from their productive societies and even more money went to prop up corrupt, tyrannical or inefficient governments.

Wall Street Journal. On looking down from the Parthenon one sees nothing, more precisely one sees a brown-pink layer of haze that covers all of Athens.

China One-Child Policy. The following numbers are approximate. Each professor in China makes about $ 100 per month as I learned in June 1986, but rent of his 2-bedroom apartment costs only $ 5 and the rest is spent on food, clothing, and saving money for a new bicycle. There are almost no cars and red lights are regarded as "yield" signs.

When a couple has one child their salary is increased by about 10% (to $ 110). If and when the couple has a second child, the salary goes down by about 10% (to $ 90). The policy is endorsed by the population, who realize that

(1) their number will still increase for the next 30 years because of the large number of young people,
(2) it finally will go below 900,000,000 by the year 2050, and
(3) the Chinese race, with their vast natural resources will thereby survive.

On the other hand, the population of Mexico City is over 15,000,000 and is increasing very exponentially. The automobile traffic and pollution is considered the worst in the world.

Graph theoretic models

There are exactly 11 graphs with four nodes as shown in Figure 1,

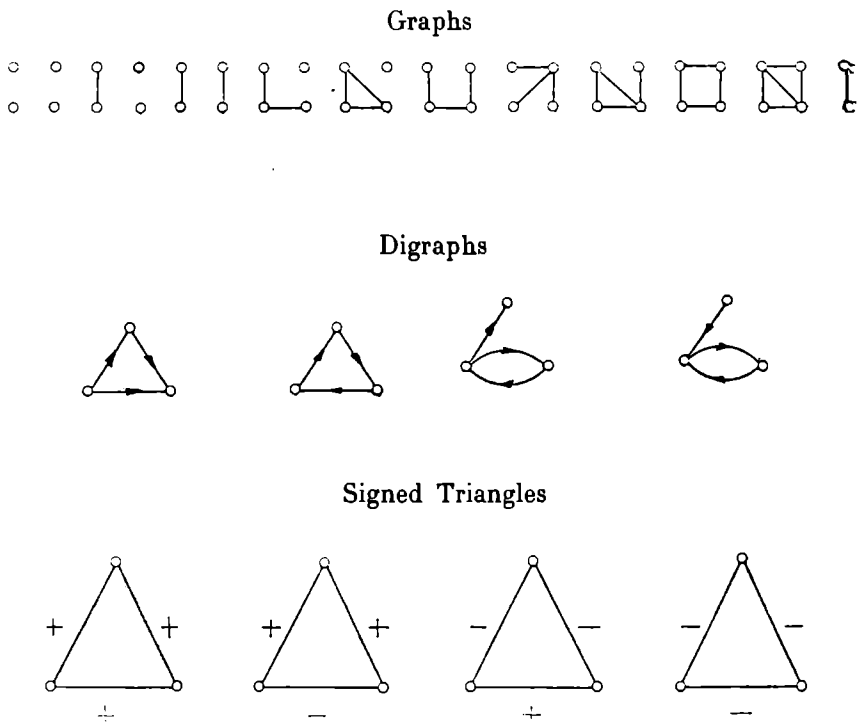

Figure 1. Graphs, digraphs, signed graphs and networks.

along with the four digraphs having three nodes and three arcs, and the four signed triangles.

Graph theory applies as a logical mathematical model to all subjects in which structure is present either explicitly or implicitly. Thus there are useful and insightful graph models in (amongst other disciplines)

(a) geography: places and direct connection
(b) chemistry: atoms and chemical bonds
(c) psychology: people and relationships
(d) electrical engineering: junctions and resistances
(f) business: interlocking directorates
(g) the quality of life: variables and causations

A mathematical model is a tentative logical hypothesis for the representation of real world situations. As long as it is correctly predictive, it is retained. When it fails to predict the observed facts, whether physical, social, educational, political or other, it must be either modified or replaced. We offer three such models, all based on graphs and their matrices, as relevant for the study of resources.

Flow of resources

The flow of resources can be realistically modeled by capacitated networks which are graphs or digraphs with numerical weights on their edges or (directed) arcs. These wights can denote costs or distances or time required for that step in the system's process, or capacity which tells the number of items which can flow in that channel per time unit. (In electrical networks, for example, the weight is the current, while in structural chemistry, the topology (graph) of a molecule has edge weights which tell the strength of the chemical bond).

Flows in networks have as their central theorem and algorithm the work of Ford and Fulkerson (1948). Their result states that:

The maximum possible flow through a network N from its sources to its terminus t equals the minimum total capacity of a cut set of edges (or arcs) whose removal from N dissrupts all communication from s to t.

This theorem was anticipated by Menger (1927) as reported in the book on graph theory by Harary (1969). An elementary introduction to graphs and graph modelling is given in Hage and Harary (1983) who wrote for a readership of anthropologists a more accessible book which does not require a powerful mathematical background.

Signed digraphs and the environment

The impact of variables on each other was pioneered by Roberts (1976) in his work on the environment using directed signed graphs (discovered by Harary, 1953). Here the nodes are statistical variables u.v. ... and there is an arc uv from u to v when variable v depends on u. This arc is positive (or negative) if when the value of u increases, that of v increases (or decreases). In the theory of Roberts, the presence of a positive directed cycle is highly unstable. An example is shown in Figure 2.

Decision making concerning a new product in the market

It is sometimes possible to make an early correct decision concerning the prospects for success of a new product in the market place. The examples which were first considered were a new brand of coffee or a toothpase. However the analytic procedure should apply as well to the prospects for larger items such as (a) a new car, (b) a new family of computers, (c) a new educational curriculum, or (d) a new economic policy. These four items were specifically named because they are vividly illustrated by some of the most disastrous failures of new products in recent decades:

(a) The Ford Motor Co. lost more than one billion dollars when the Edsel car did not catch on.

(b) The AT & T corp. also lost more than one billion dollars when their heavily advertised line of computers did not sell just two years ago.

(c) In the USA at least, the so-called "new math", which taught axiom systems to small children, was a dismal failure. Replacement curricula are still under study.

(d) Supply side economics has contributed mightily to the recent fact that the USA has become a debtor nation.

In an article, "Brand loyalty and brand switching", Harary and Lipstein (1961) proposed an algorithm for early detection of the likelihood of success of a new product:

(1) Begin by collecting a reliable sample of empirical data which tells the products bought by the same consumers in each time period.

(2) Rewrite these data in the form of probability matrices with non-negative entries and every row summing to 1. Call these matrices P_1, P_2, P_3, ... for the consecutive time periods, with matrix P_1 showing the proportions of brand loyalty (the diagonal entries) and brand switching (off-diagonal) from time 0 to time 1, etc.

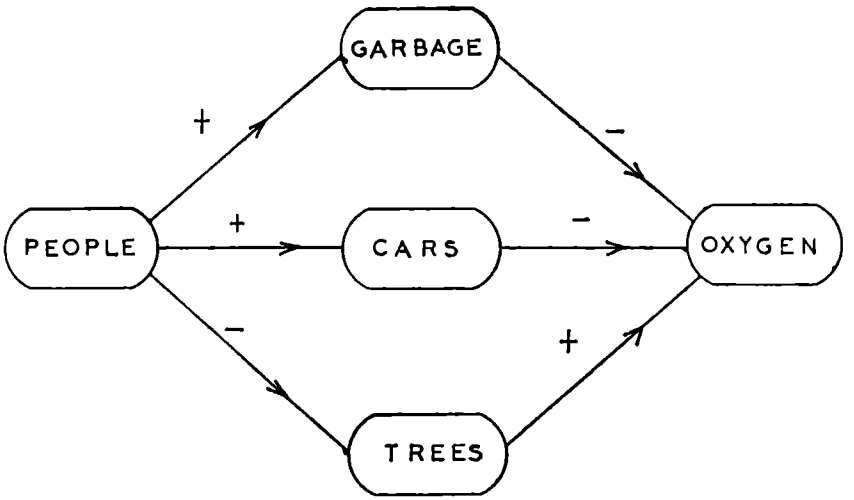

Figure 2. Impact of environmental variables, after Roberts.

(3) Calculated the "change matrices" C_1, C_2,... defined by $P_1C_1 = P_2$, $P_2C_2 = P_3$,...

(4) Evaluate the largest given value of the matrices Ci after they stabilize.

If this value is less than 1, the new product will likely not succeed.

If it is greater than 1 it probably will succeed.

If it is approximately 1, more observations are needed.

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