NIKOLAOS GEORGOPOULOS – PRODROMOS EFTHYMOGLOU –
JIANNIS LAZARIDIS

HOW SUBJECTIVE PROBABILITIES CAN BE SET FOR HELPING THE DECISION–MAKER TO TAKE THE CORRECT DECISION

PLAN

ABSTRACT

It is important in many areas of management to provide a measure of uncertainty. The only effective way to measure uncertainty is to use a numeric measure such as probability. The phrases used by some managers such as possible, doubtful, etc., are of little value as they mean different levels of uncertainty to different people. Probabilities being numeric in nature can be used to extend the knowledge of the situation by using the various rules of probabilities including the more complex such as Bayes's Theorem. When theoretical models are not available, probabilities can be assessed subjectively. An assessment of probability reflects an individual's state of information about a given quantity or event. The personal interpretation of probability is the corestone of the decision analysis philosophy. This paper presents the philosophy and practise used in probability encoding.

1. INTRODUCTION

Probability, in general, is used to describe the chance of outcome in an uncertain event. When we say "subjective probability", we mean a probability that has been set by individual or a group of individuals using his, her or their collective experience and knowledge due to fact that there are insufficient data to allow the probability to be calculated in the classical manner (Carter 1984c).

That is, a "subjective probability" is a quantification of personal uncertainty. It is characterized by a number between "zero" and "one", representing an individual's degree of belief in an outcome of an uncertain event. The assessment "zero" indicates a belief that the event is impossible and "one" indicates a belief that it is certain.

The theory of subjective, or personal probability, plays an important role with respect to numerous inferential and decision-making models. In particular, Bayesian approaches to inferential problems often involve subjective probabilities, and an important input to decision analysis models is a set of subjective probabilities representing the judgments of a decision maker or of an expert consulted by a decision-maker.

Partly as a result of the increasing interest in Bayesian inference and decision analysis, the elicitation of subjective probabilities has received a considerable amount of attention in recent years (Carter 1985b). Various methods have been involved to aid an individual in assessing (encoding) subjective probabilities to be used in inferential and decision-making situations. The methodology and experimental work regarding probability elicitation has ranged from investigations of details of the actual elicitation procedures (e.g. comparisons of different response modes) to studies of the relationship of the elicitation process to the broader framework of modeling in decision analysis.

Methods for assessing probabilities can be placed in two categories. One suggests asking for a probability directly. For instance, a marketing manager might be asked directly for the probability that sales exceed a certain level. The other recommends that probabilities be obtained indirectly from a series of choices. By making choices between two uncertain events, a probability can be deduced. The assessment problem occurs because individuals have feelings about probabilities that are not usually directly expressed in terms of numbers. That is, we may have definite feelings about the probabilities associated with an uncertain event, but not have these feelings "coded" in terms of probabilities.

So, when subjective probabilities are set for use in decision analysis, two issues can be raised. One is the question of how these probabilities can be obtained. The various procedures are aimed at determining probabilities that provide a "correct" representation for the uncertain event. The second is the question of what factors we have to take into consideration, because of the way that individuals are respond in questions about uncertain quantities. The following discussion is based on these two issues.

2. HOW SUBJECTIVE PROBABILITIES CAN BE OBTAINED

2.1. Betting Odds

Subjective probabilities can be considered betting odds (Carter 1984b). That is, they can be treated just like the probabilities that the decision maker would desire in a lottery situation of his or her own design in which the payoffs are identical in every respect to the possible payoffs from the actual decision being evaluated. For example, suppose that a contractor assigns a subjective probability of 0.5 to the event of winning a contrast that will increase profits by $50,000 and that losing the contract will cost $10,000.

This contractor ought to be indifferent between preparing a bid for the contract and gambling on a coin toss where a head provides a $50,000 win and a tail results in a $10,000 loss. The subjective probability for winning the contract can therefore be transformed directly into an "objective" 0.5 probability of obtaining a head from a coin toss, we can then substitute the latter into the decision analysis.

Furthermore, a practical benefit of substituting a hypothetical gamble for an actual uncertainty is that subjective probabilities can be used in conjunction with the traditional long-run frequencies of occurrence.

2.2. Assessing Methods and Modes that the Individual Response

After accepting the premise that judgment about uncertain events can be expressed as probabilities, we still have the problem of making the assessments. The whole process of extracting and quantifying individual judgment about uncertain quantities plays an important role in the application of decision analysis. It is usually performed in the context of a specific decision problem. However, the general benefit of this process extends beyond the analysis of specific decisions. It improves a subject's awareness of his state of information and provides a clear means for communication and inference about uncertainty.

Most assessing methods for subjective probabilities are based on questions for which the answers can be represented as points on a cumulative distribution function (Tydeman and Mitchell 1978). The different assessing methods used, vary according to whether they ask a subject to assign probabilities, values, or both. Any of the assessing procedures consists of a set of questions that the subject responds to either directly by providing numbers of indirectly by choosing between simple alternatives or bets.

In the "direct response mode", the subject is asked questions that require numbers as answers. Depending on the method being used, the answers will be given in form of either values or probabilities.

In the "indirect response mode", the subject is asked to choose between two or more bets (or alternatives). The bets are adjusted until the subject is indifferent to choosing between them. This indifference can then be translated into a probability or value assignment. When an external reference process is used, one bet is defined with respect to the uncertain quantity and the other with respect to a familiar reference event. Another procedure is to ask the subject to choose between events defined on the value scale for the uncertain quantity, where each event represents a set of possible outcomes for the uncertain quantity. This types of response mode uses internal events for comparison.

2.3. Assessing Approaches

According to the assessing method and response mode used, there are two basic approaches for assessing subjective probabilities: Indirect response approach and direct response approach (Carter 1982). Each of them uses different techniques for assessing subjective probabilities. We will analyze these different techniques that we use in each approach. In effect, after the analysis we will be able to understand how the subjective probabilities are obtained and how we assess them for using in the decision analysis.

2.3.1. Indirect Response Approach

2.3.1. A. Probability wheel

One of the most useful tools that has been discovered for assessing indirect responses from subjects in the probability wheel (Matheson and Winkler 1976). It is consisted of a disk with two adjustable sectors, on blue and one red or orange, with a fixed pointer in the center of the disk. When spun, the disk will finally stop with the pointer either in the blue of the orange sector and thereby the probabilities of the pointer indicating either sector when the disk stops spinning.

The subject is asked which of the two events he thinks more likely–the event relating to the uncertain quantity (for example, the event that next year's production will not exceed X units), or the event that the pointer ends up in the orange sector. In the following, the proportion of the colors on the disk can be changed until the subject will find the two events equally likely. The relatively amount of orange is then assigned as the probability of the event.

One of the advantages of the probability wheel is that the probability can be varied continuously from "zero" to "one". However, because it is very difficult for persons to distinguish between the sizes of very small sectors, we use the probability wheel to evaluate probabilities in the range from 0.1 to 0.9. Another tool similar to probability wheel is the horizontal bar with a movable marker which define two events–one to the left and one to the right of the marker (Kahneman and Tverskey 1972).

Another alternative is to ask the person to visualize an urn with standard number of balls of two colors (e.g. blue and orange) (Lapin 1981). A ball is drawn at random and the reference event is that the ball will be orange. The composition of the urn then can be changed until it reflects the probability of the event in question.

For all these alternatives, it is preferred to use the probability wheel, because subjects understand better this method than the others.

2.3.1. B. Interval technique

In the beginning of this technique, an interval is split in two parts. The person is asked to choose one of the parts he thinks is the most likely to occur (or which part he would prefer to bet on). Then the dividing point is changed to reduce the size of the part considered most likely and the person is asked again to choose.

This continues until the person is indifferent between the two parts. So, each of these parts are assigned equal probabilities. Beginning with an interval covering all possible outcomes and then splitting into two subintervals first gives the median then the quartiles and so on.

The procedure stops after the quartiles have been obtained, because each question depends on earlier responses, and the errors are thus compounded. The interval technique can also be used by splitting an interval into three or more parts at a time.

2.3.1. C. Relative likelihoods

In this technique the subject is asked to evaluate relative likelihoods (or odds) to two well–defined events (Halloway 1979). For example, the subject first is asked whether he thinks next year's sales more likely to be above or below a standard quantity (say 1,000 units). Then he is asked how many times it is more likely to occur. The relative likelihoods method is used mainly for uncertain quantities that might have only few outcomes.

2.3.2. Direct Response Approach

2.3.2. A. Cumulative Probability and Fractiles

In this technique the subject might asked to evaluate a cumulative probability for a given value —e.g. what is the probability that next year's profits will be less than or equal to $10,000?— or to evaluate the value for a given probability —e.g. what is the level of profits that corresponds to a 20 percent probability? Then the probability can be expressed as absolute number (0.20), as a percentage (20 percent), or as a fraction (one in five). Mainly, for small probabilities the last way is used because people usually can distinguish more easily, between say, "one in 100" and "one in 1,000", that between 0.01 and 0.001. In general the form that will be used depends on the formality of the subject with the particular form.

2.3.2. B. Graphs

In this technique the subject either draw a density function or a cumulative distribution or state a series of pairs of numbers (value and probability). Another method is to show the subject a series of density functions and then the person is asked to choose the one that is most closely to his judgment (Carter 1984d). Additionally density functions can be generated by taking a family distributions and varying the parameters.

2.3.2. C. Verbal Assessing

Here there are two phases. In the first phase, descriptors such as "high", "medium" and "low" production cost is used. Quantitative interpretation of the descriptors is the encoded in the second phase. This method could be of particular use in dealing with quantities that have no ordinal value scale.

In the above analysis we have examined the most used techniques in assessing (encoding) subjective probabilities. In general, we can say that subjects seem to fall into two categories: those who feel capable of giving direct numerical probability assignments and those who have difficulties in making such judgments. Most people are in the second category. Additionally many individuals who prefer direct numerical responses are later found to have little confidence in their initial numerical responses.

For that, until a person is trained to think about probabilities the indirect approach is better. Of these techniques the probability wheel is the most used one (Lapin 1981). However, there is some evidence that, after an individual becomes an expert on assessing probabilities, the direct approach is effective.

3. MODES OF HUMAN JUDGMENTS

Except the procedures that they are used for collecting subjective probabilities, we have to take into consideration the way that individuals respond in questions about uncertain quantities (Tverskey and Kahneman 1973). That means that psychologic factors have to be taken into account. This is the debate of the second part of this discussion.

For the purpose of this discussion, the subject is assumed to have an underlying stable knowledge regarding the quantity under investigation. This knowledge may be changed by receiving new information. The task of the decision-analyst is to elicit from the subject a probability distribution that describes his underlying knowledge. Conscious or subconscious discrepancies between the subject's responses and an accurate description of his underlying knowledge are termed biases.

3.1. Motivational Biases

Understanding how people think about uncertainty can provide important information on how assessments should be made (Slovic and Litchenstein 1971). As the first step "motivational" biases should be identified. Motivational biases are either conscious or subconscious adjustments in the subject's responses motivated by his perceived system of personal rewards for various responses.

In other words, the person may want to influence the decision in his favor by giving a particular set of responses. Furthermore he may want to bias his response because he believes that his performance will be evaluated by the outcome. The classic example is asking salespersons to predict sales in their territory for the next year. If these predictions are to be used for setting goals, quotas and incentive schemes, there may be clear biases, that will work to the advantage of the salesperson. When "motivational" biases exist, the design of incentive systems that control of eliminate the biases can be challenging.

3.1.1. Scoring Rules

One class of incentive schemes that has been studied is scoring rules. A "proper" scoring rule is one which encourage an assessor to reveal his true opinions and to make his stated probabilities correspond with his judgments. Scoring rules, which involve the computation of a score based on the assessor's stated probabilities and on the event that actually occurs, are useful in the evaluation of probability assessors as well as in the elicitation process itself.

In terms of elicitation, the role of scoring rules is to encourage the assessor to make careful assessments and to be "honest", whereas in terms of evaluation, the role of scoring rules is to measure the "goodness" of the probabilities.

The development of scoring rules has, in general, been restricted to the elicitation of individual probabilities or discrete probability distributions. In many situations of course, the variables of interest are discrete, or discrete approximation can be used.

3.2. Cognitive Biases

Even when a subject is honest —in terms of motivational biases— he may still have cognitive biases. Cognitive biases are either conscious or subconscious adjustments in the subject's responses that are systematically introduced by the way the subject intellectually processes his perceptions (Warr 1977). Since cognitive biases may stem from a particular mode of judgment, an important responsibility of the decision–analyst is to discover what modes of judgment the subject might be using and then try to adapt the whole discussion to minimize biases. There have been defined several different modes of judgment. The most important of these are the following:

3.2.1. Representativeness

Representativeness means that the probability of an event or a sample is evaluated according to the degree to which it is considered of the process of population from which it originated. When judging the probability of an event by representativeness, one compares the essential features of the event to those of the structure from which it originates. In this manner, one estimates probability by assessing similarity or connotative distance (Kahneman and Tverskey 1973).

Furthermore, there is a strong tendency to place more confidence in a single piece of information that is considered representative than in a larger body of more generalized information. For example, suppose that a production manager is trying to evaluate a new process. A test indicates poor performance. Although the same process has not been used before experience with other similar processes indicates that its should work. Additionally, initial tests often provide spurious results. There is a tendency to rely excessively on the poor test results with the specific process (i.e. the most specific representative information) rather than including the more general information. This type of bias can often be reduced by structuring the assessment problem to explicit include assessment of prior probabilities using the general information followed by the use of Bayesian revision to take into account new information.

Another problem of this technique is that of insensitivity to sample size. Kahneman and Tverskey (1972) proved that when subjects were presented with a problem that involved sample size, this was totally ignored. One of the problems they used was concerned with the sampling of a large group of men (average height 5′ 10′′, samples of 10, 100, and 1,000 men) and then using these to obtain average height figures. The people were asked to estimate the probability of obtaining samples with average heights in excess of six feet. The majority of people arrived at the same figure for all three samples sizes which showed a lack of understanding of the fact that the greater the sample size the smaller the variation in means and therefore the probability of such an event should reduce as sample size increases.

3.2.2. Availability

Availability refers to the ease with which relevant information is recalled or visualized (Halloway 1979). Probability assignments are based on information that the subject recalls or visualizes. So the probability of a breakdown in a production process may be assigned by recalling post breakdowns. Information that made a strong impression because of the consequences associated with it and recent information are examples of information that is readily available and hence often given too much weight. For example if one was asked to assess the risk of fatality due to motor vehicle accidents, one way would be to bring in mind the fatalities encountered and use this to assess the risk.

Additionally, imaginability plays an important role in the evaluation of probabilities and can be important in mathematical concepts. For example if you were to ask a production manager the number of possible schedules it is possible to draw up by putting 40 different jobs through a five machine schedule he/she is most likely to underestimate the possible number. This lack of imaginability of certain mathematical concepts can cause problems in the setting of some subjective probabilities.

Availability appears to be important mode of judgment in most probability encoding sessions. Furthermore deliberate attempts to make available competing information by asking about how past outcomes can reduce the bias.

3.2.3. Adjustment and anchoring

The most readily available piece of information often forms as initial basis for formulating responses; subsequent responses then represent adjustments from the basis. An example of this could be found in the forecasting field. The initial forecast will be set by using mathematical forecasting, this being the anchoring stage. This value can then be adjusted to take into consideration other factors like advertising, the present economic climate, the competitors' products etc., this being the adjustment stage.

In some instances, adjustment from the initial point is insufficient. Anchoring results when subsequent points on a distribution are not processed independently from the starting point. An experimental result often observed is that subjects who are first asked for a median or most likely point, fail to adjust adequately. The result is a control bias.

3.2.4. Unstated Assumptions

Typically, a subject's responses are conditional on various unstated assumptions. Consequently, the resulting probability distribution does not properly reflect his total uncertainty.

When unexpected outcomes occur, assessors sometimes "explain" them by pointing to some underling assumption that was violated. For example, in assessing the sales for a new product an individual may make a series of implicit assumptions about the economy, competitors, and the production capability of his own organisation.

While the subject cannot be held responsible for taking into account all possible eventualities that may affect the quantity he is assessing, it is his responsibility to state the assumptions he is making about his own limits of responsibility. Good assessment practice makes explicit the important assumptions on which the probabilities are conditioned.

3.2.5. Coherence

People sometimes appear to assign probabilities to an event based on the ease with which they can fabricate a "plausible" scenario that would lead to the occurrence of the event (Kahneman and Tverskey 1972). The event is considered unlikely if no reasonable scenario can be found; it is judged likely if many scenarios can be composed that could make the event occur or if one scenario is particularly coherent.

The credibility of a scenario to a subject seems to depend more on the coherence with which its author has spun the table than on its intrinsically "logical" probability of occurrence. For example, the probability assigned to the event that sales would exceed a high volume may depend on how well market researchers have put together scenarios that would lead to that volume; these could be scenarios on what markets might be penetrated and what the penetration rate might be with a reasonable marketing effort.

It is thus important that the discussion of scenarios leading to possible outcomes for an uncertain quantity be well balanced, since the relative coherence of various arguments can have a strong affect on the probability assignments.

4. CONCLUSION

When theoretical models or empirical evidence is not available, probabilities can be assessed subjectively. The personal interpretation of probability is the corestone of the decision analysis philosophy. An assessment of probability reflects an individual's state of information about a given quantity or event. Since various people are likely to have different information, two persons can arrive at different probability assignments for the same uncertain quantity.

Two questions arise when we want to set subjective probabilities for use in decision analysis. How these probabilities can be obtained, what factors we have to take into account because of the way that individuals are respond in questions about uncertain quantities.

For the first question, there are some techniques that are used. Each probability assessing technique can be classified according to the assessing method and response mode used. The answer to the second question involves the psychology of human information processing.

The decision–maker is the person (or group of persons) who has responsibility for the decision under consideration. It follows that a decision analysis must be based on the decision maker's beliefs and preferences. He may be willing to designate some other persons as his expert(s) for assessing the uncertainty in a particular variable if he feels that the expert has a more relevant information base. The decision–maker can then either accept the expert's information as his input to the analysis or modify it to incorporate his own judgment.

Whatever the decision–maker does, he/she has to take into consideration the above questions. If he/she does take these questions into account, then the subjective probabilities discussed in this paper may help him/her to make the correct decisions.

REFERENCES

Carter M. P. (1982), "An Examination of the Principal Myths Surrounding Management Information", *West Midlands Regional Management Centre Review*, 2, No 1.

Carter M. P. (1984a), "Costing Management Information –a More Formal Approach", *Journal of Information Science*, 9.

Carter M. P. (1984b), *The Cost of Management Information*, Management Decision, 22, No. 5.

Carter M. P. (1984c), "The Measurement of Uncertainty for Use in Management Decision Analysis", *West Midlands Regional Management Centre Review*, 3 (2).

Carter M. P. (1984d), "Some Practical Aspects of Management Decision Analysis", *West Midlands Management Centre Review*, 3 (2).

Carter M. P. (1985a), "The Valuing of Management Information. Part I: The Bayesian Approach", *Journal of Information Science*, 10.

Carter M. P. (1985b), "The Valuing of Management Information. Part IV: A Practical Approach", *Journal of Information Science*, 10.

Hampton J. M., Moore P. G. and Thomas H. (1973), "Subjective Probability and its Measurement", *Journal of the Royal Statistical Society*, Vol. 136.

Holloway C. (1979), *Decision Making Under Uncertainty*.

Kahneman D. and Tverskey A. (1972), "Subjective Probability, A Judgment of Representativeness", *Cognitive Psychology*, Vol. 3.

Kahneman D. and Tverskey A. (1973), "On the Psychology of Prediction", *Psychological Review*, Vol. 80.

Lapin L. (1981), *Quantitative Methods for Business Decisions*, 2nd edition.

Matherson J. and Winkler R. (1976), "Scoring Rules for Continuous Probability Distributions", *Management Science*, June.

Slovic P. and Lichtenstein S. (1971), *Organisational Behaviour and Human Performance*, Vol. 6, pp. 649.

Tverskey A. and Kahneman D. (1973), "Availability: A Heuristic for Judging Frequency and Probability", *Cognitive Psychology*, Vol. 5.

Tydeman J. and Mitchell R. (1978), *Subjective Future for Decision Making*, Research Monograph Series, No. 3, Administrative Studies Program, Australian National University.

Warr P. (1977), "Aided Experiments in Social Psychology", *Bull. Br. Psychology Soc.*, No. 30.