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TESTING NONLINEARITIES WITH BDS–STATISTIC

PLAN

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ABSTRACT

The purpose of this article is to investigate whether monthly changes in the General Index of the Athens Stock Exchange contain any nonlinearities. The methodology follows the lead of Brock, Dechert, and Sheinkman statistics (BDS), based on the notion of the correlation integral. The null hypothesis is that the data are independently and identically distributed (iid). This test is applied instead of an attempt to determine if the data are stochastic or chaotic.

1. INTRODUCTION

Is the current market price of an asset the best estimate to the future price, making all attempts at forecasting price movements a useless exercise? Or is it possible to develop a model to fully explain month-to-month price fluctuations? The answer probably lies somewhere between these two extremes, in improving existing examples of short-term forecasting success. Many empirical studies have shown that stock returns follow a random walk model and are unable to be forecast successfully, but new evidence has been presented that shows that these results may be supplemented, and perhaps replaced, by a new hypothesis of an underlying nonlinear process.

Time series generated by these systems all exhibit little or no serial correlation, and yet the series x_t (t=1,2,...,T) is not stochastically independent of x_{t-1}. Thus, traditional tests of linear dependence (such as autocorrelation coefficients, run test, etc.) will not detect the nonlinear dependence.

Nonlinear processes are the focus of research in many scientific areas. Although the beginings of these new techniques have been in the natural sciences of physics, they are now being applied to economics. But is only recently that these methods are applied in finance.

Scheinkman and LeBaron (1989) found that a significant part of the variation in the weekly value-weighted portfolio of the Center for Research in Security Prices (CRSP) stock index returns is due to nonlinearities instead of randomness. Brock, Hsieh and LeBaron (1992) found nonlinear dependence on the CRSP and S&P 500 index continuously compounded weekly returns. Hsieh (1989) and Hsieh (1991) found substantial evidence of nonlinear dependence in daily exchange rate changes and weekly returns of CRSP from 1963 to 1987 respectivelly. Kugler and Lenz (1993), they also detect nonlinearities in the returns of ten currencies against the US dollar. Peters (1989,1991) provide additional evidence that the S&P 500 index has an underlying low dimensional chaotic attractor. Sirlantzis and Siriopoulos (1993) found biased random walk and low dimensional chaos in the monthly returns of the General Index of the Athens Stock Exchange (ASE) using the Grassberger and Procaccia (1983) method. Other studies in this area include Hinish and Paterson (1985), Larrain (1991), etc.

The purpose of this article is to investigate whether changes in montly returns of the ASE exhibit nonlinear dependence, using a method proposed by Brock, Dechert and Scheinkman (hereafter BDS, in Brock, Hsieh and LeBaron 1992 op.cit.). Section 2 briefly discuss nonlinear dynamics and the methodology used in this article. Section 3 presents an application to ASE, and section 4 concludes the paper with a discussion of the results and future research.

2. RESEARCH METHODOLOGY

An efficient way to test for chaos is to consider the following quantity, called the correlation integral, studied by Grassberger and Procaccia (1983), which is a measure of spatial correlation of scattered points or particles in m-dimensional space:

$$C_{m,T}(r) = \sum_{t<s} I_r\left(x_t^m, x_s^m\right) \times \left[2/T_m\left(T_m - 1\right)\right] \quad (1)$$

where, $T_m = T-(m-1)$, $x_t^m = (x_t, \ldots, x_{t+m-1})$, $t=1, 2, \ldots, T$ and $I_r(x_t^m, x_s^m)$ = indicator function that equals unity if $\|x_t^m - x_s^m\| < r$, and zero otherwise, m is the embending dimension; $\|.\|$ = the norm as measured by the Euclidian distance.

This quantity measures the fraction of pairs of points of $\{x_t\}$ that are within a distance of r from each other. For stochastic and deterministically chaotic systems, as $T \to \infty$:

$$C_{m,T}(r) = C_m(r) = \text{Prob}\{\|x_t^m - x_s^m\| < r\} \quad (2)$$

The correlation integral is used by Grassberger and Procaccia to define the correlation dimension of $\{x_t\}$:

$$d_m = \lim_{r \to 0} \{\log[C_{m,T}(r)]/\log(r)\} \quad (3)$$

if the limit exist.

The correlation dimension is used to differentiate between deterministic chaos and stochastic systems. If chaos is present, as d is estimated for increasingly larger values of the embending dimension, the dimension estimate will stabilize at some value. If the stabilization does not occur, the system is considered «high–dimentional» or stochastic.

However, different definitions of chaos exist in the literature. All of them attempt to capture the notion of sensitive dependence upon initial condition (SDIC). A measure to capture the notion of SDIC is the largest Lyapunov exponent (λ_i). For an application to monthly returns of the ASE, see Sirlantzis and Siriopoulos (1993).

In this paper, instead of trying to distinguish a chaotic system from a stochastic system, we apply an alternative strategy proposed by BDS. BDS propose to test the null hypothesis that the data are independently and identically distributed (iid), using a statistic based on the correlation integral. This statistics is (Brock, Hsieh and LeBaron, op.cit., ch. 2):

$$W_{m,T}(r) = T^{1/2} \times [C_{m,T}(r) - C_{1,T}(r)^m] / \sigma_{m,T}(r) \quad (4)$$

BDS statistics has a limiting standard normal distribution under the null hypothesis of iid. Large values would indicate strong evidence for nonlinearity in the data.

3. AN APPLICATION TO THE ATHENS STOCK EXCHANGE (ASE)

The data consist of the monthly closing prices of the General Index of the ASE, from January 1980 to February 1993 (158 observations). To ensure that the data are stationary, the first difference of the log is taken:

x_t = [(log of the ASE index on month t) – (log of the ASE index on month t–1)] (5)

In computing the BDS statistics we have two important issues to deal with: the choice of r and m.

Table 1 reports a summary statistics of the data, and Table 2 presents the results of the BDS statistics for m = 2, 3, 4, 5, 6, 7, 8, 9, 10 and r = 0.147, 0.01225, 0.049, 0.0735, 0.098 according to Hsieh (1989), or r = 0.064 according to Wolf et al (1985) 10% of the range between the maximum and the minimum value of the data rule of thumb. The critical value for the test is 1.96 for 5% level of significance.

Note that all of the BDS statistics lie in the positive tail of the standard normal distribution. So, the data reject the null hypothesis of iid, with only three exeptions.

4. CONCLUSION AND ADDITIONAL EVIDENCE

This study presents BDS test for the existence of deterministic chaos in the monthly values of the General Index of the Athens Stock Exchange. Using BDS statistics we reject the null hypothesis of iid. This result is compatible with other previous tests.

In a recent work (Sirlantzis and Siriopoulos 1993) we confirmed the long memory effect of the ASE index by two separate types of nonlinear analysis: i) The application of the R/S analysis found a biased random walk with a memory length of about 36 months. ii) The largest Lyapunov exponent turned to be positive with a value of about 0.0277, that is 0.0277 bit of information lost per month (1/0.0277 = 37 months).

In general, all the tests suggest nonlinear dependence and low dimensional chaos. As a next step, models of short–term returns determination should be developed.

TABLE 1
Data Description

# of Obs.	=	158			
Maximum	=	0.2865			
Minimum	=	-0.4097			
Spread	=	0.6961			
Average	=	-0.00165			
Std. Dev.	=	0.00984			
SD/Spread	=	0.1414			
Regression on it:					
			Coefficient of t	=	0.000088
			t-statistic	=	0.000191
			R squared	=	0.00076

TABLE 2
BDS–Statistics

r	*m*	*ASE*
1.50	2	3.27
1.50	3	4.18
1.50	4	5.14
1.50	5	6.59
1.50	6	7.64
1.50	7	8.80
1.50	8	9.97
1.50	9	11.30
1.50	10	12.80
1.25	2	3.38
1.25	3	4.12
1.25	4	5.10
1.25	5	6.52
1.25	6	7.41
1.25	7	8.43
1.25	8	9.35
1.25	9	10.40
1.25	10	11.72

TABLE 2
BDS–Statistics (continued)

r	*m*	*ASE*
1.00	2	3.61
1.00	3	4.38
1.00	4	5.29
1.00	5	6.67
1.00	6	7.31
1.00	7	8.20
1.00	8	8.80
1.00	9	9.72
1.00	10	11.40
0.75	2	3.79
0.75	3	4.78
0.75	4	5.28
0.75	5	6.58
0.75	6	7.02
0.75	7	7.83
0.75	8	9.44
0.75	9	11.38
0.75	10	13.74
0.50	2	3.70
0.50	3	5.67
0.50	4	5.29
0.50	5	6.34
0.50	6	6.48
0.50	7	5.36
0.50	8	-1.33
0.50	9	-1.04
0.50	10	0.82
0.069	2	3.74
0.069	3	5.03
0.069	4	5.67
0.069	5	7.24
0.069	6	7.86
0.069	7	7.92
0.069	8	8.24
0.069	9	12.23
0.069	10	15.80

r: multiples of index standard deviation, *m:* embedding dimension, *ASE:* the BDS test for iid conditions in the logaritmic returns of the General Index of the Athens Stock Excange.

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