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MODEL SPECIFICATION AND FORECASTING ACCURACY OF SOME SEASONAL INTERACTION MODELS: AN EMPIRICAL INVESTIGATION USING THE "AIRLINE" DATA

PLAN

Abstract

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ABSTRACT

Two well know approaches to seasonal forecasting, are the Holt-Winters and Box-Jenkins methods. Forecasts obtained by these methods, are compared with the forecasts from some seasonal interaction models (transfer-function type ARIMA models employing trend and seasonal dummy variables). The advantages of each method can be judged from their goodness of fit and their ex-post forecasting accuracy in the particular case of the "airline" data.

Key Words: Holt - Winters, Sarima, Seasonal Interaction, Forecasting.

INTRODUCTION

Time series are usually decomposed into trend (T), cyclical (C), seasonal (S) and irregular (I) components and these effects are assumed additive, multiplicative, or mixed.

Two well know approaches to short-term seasonal forecasting are: a) the Holt-Winters three parameter linear and seasonal exponential smoothing and b) the Box-Jenkins approach (SARIMA models) where ARMA models are fitted to previously transformed and differenced data when this is necessary. A third approach is the use of seasonal interaction models (transfer-function type ARMA models with trend and seasonal dummy variables). Because there are many possible specifications of these models, some of which are described below, we apply them next to the "airline" data. From the goodness of fit and the comparison of their ex-post forecasting accuracy with those of the other methods, we can drow conclusions about their suitability in each particular case. It turns out that a four stage process similar to the one suggested by Box and Jenkins (i.e. identification, estimation, diagnostic checking and forecasting) must be followed in this case also. Next we describe briefly these methods.

THE HOLT - WINTERS APPROACH TO SEASONAL FORECASTING

The Holt - Winters revision equations for series with linear trend and multiplicative seasonality are:

$$L_t=a(X_t/I_{t-p}) + (1-a)(L_{t-1}+T_{t-1})$$

$$T_t=\beta(L_t-L_{t-1}) + (1-\beta)T_{t-1}$$

$$I_t=\gamma(X_t/L_t) + (1-\gamma)I_{t-p}$$

The forecasts are computed from:

$$X_t(m)=(L_t+mT_t)I_{t-p+m} \qquad m=1, 2, ..., p$$

The additive version is:

$$L_t=\alpha(X_t-I_{t-p}) + (1-\alpha)(L_{t-1}-T_{t-1})$$

$$T_t = \beta(L_t-L_{t-r}) + (1-\beta)T_{t-1}$$

$$I_t=\gamma(X_t-L_t) + (1-\gamma) I_{t-1}$$

The forecasts are computed from:

$$X_t(m)=L_t+mT_t + I_{t-p+m} \qquad m=1, 2, ..., p$$

where: X_t= actual series, L_t= local mean level, T_t=trend,

I_t= seasonal index and p= length of seasonality.

Notes:

1. An improved model assumes damped trend. See ref. (6).
2. The additive version can be applied to the (log) tranformed data.
3. Analysis of the residuals after fitting these models, can show any patterns remaining, but it doesn't exist any method for correcting this.

THE BOX - JENKINS APPROACH TO SEASONAL FORECASTING

The general class of multiplicative Sarima models: (P,d,q) (P,D,Q) has the form:

$$\varphi(B)\,\Phi(B^s)\,\nabla^d\,\nabla_s^{\,a}\,X_t^{(\lambda)} = \theta_o + \theta(B)\,\theta(B^s)\,e_t$$

where:

e_t = white noise process (i.e. I.N.$(0, \sigma^2)$)

$B^d X_t = X_{t-d}$, $\nabla X_t = X_t - X_{t-1} = (1-B)X_t$ and $\nabla_s = X_t - X_{t-s}$

$\varphi(B) = (1-\varphi_1 B- \dots -\varphi_p B^p)$, $\Phi(B) = (1-\Phi_1 B^s - \dots -\Phi_p B^{ps})$

$\theta(B) = (1-\theta_1 B- \dots -\theta_q B^q)$, $\Theta(B) = (1-\Theta_1 B^s - \dots -\Theta_Q B^{Qs})$

θ_o = constant and

$$X_t^{(\lambda)} = \begin{cases} X_t^{(\lambda)} & \lambda \neq 0 \\ \log_s X_t & \lambda = 0 \end{cases}$$ (the Box-Cox transformation)

The main stages of B-J's approach are: Identification, Estimation, Diagnostic Checking and Forecasting. In particular, the appropriate model for the "airline" data is the (0,1,1) $(0,1,1)_{12}$ i.e. the:

$$(1-B)\,(1-B^{12})\log_e X_t = (1-\theta_1 B)\,(1-\Theta_1 B^{12})e_t$$

THE SEASONAL INTERACTION (ECONOMETRIC) APPROACH TO FORECASTING

The general form of these regression models is:

$$Y_t = d_o + d_1 T + \sum_{i=2}^{12} d_i D_{it} + \sum_{i=2}^{12} d_{i'} (D_{it} T_t) + u_t$$

where:

d_o=constant

$$D_{it} = \begin{cases} 1 \text{ if the } d_s \text{ relates to period } i \\ 0 \text{ otherwise} \end{cases}$$

T_t= 1, 2, 3, ..., T, T+1, ... and

$$u_t = \frac{\theta(B)\,\Theta(B^s)}{\varphi(B)\,\Phi(B^s)} = e_t$$

This model is also called "seasonal interaction model" see ref. (2) and is suitable for series with mixed seasonality and non-linear trend. It can be also extended in a number of ways according to the case. We consider here five specifications suitable for the airline data:

A. $$\log e_s Y_t = d_o + d_1 T + \sum_{i=2}^{12} d_i D_{it} + u_t$$

B. $$\log e_s Y_t = d_o + d_1 T + d_{i'} T^2 + \sum_{i=2}^{12} d_i D_{it} + u_t$$

C. $$Y_t = d_o + d_i T + \sum_{i=2}^{12} d'_{i'} (D_{it} T_t) + u_t$$

D. $$Y_t = d_o + d_i T + d'_{i'} T^2 + \sum_{i=2}^{12} d'_{i'} (D_{it} T_t) + u_t$$

E. $$Y_t = d_o + d_i T + d'_{i'} T^2 + \sum_{i=2}^{12} d'_i D_{it} + \sum_{i=2}^{12} d_{i'} (D_{it} T_t) + u_t$$

and u_t is taken in every case as:

1. $u_t = e_t$
2. $(1-\varphi_1 B)u_t = e_t$
3. $(1-\varphi_1 B-\varphi_2 B^2)u_t = (1-\theta_1 B^{12})e_t$

A PRELIMINARY ANALYSIS OF THE DATA

A classic example of an actual time series is the "International airline passengers: monthly totals" (in thousands of passengers) from January 1949 through December 1960 (144 observations). Plots of the actual data and their logarithms are shown in Figures 1 and 2. We observe that the series exhibit an almost linear trend and increasing seasonality (i.e. the seasonality is multiplicative). Taking logs the seasonality becomes additive.

Plots of the deseasonalised actual and transformed data are shown in Figures 3 and 4. It is clear that the trend is not linear, so models assuming linear trend in this case are misspesified. (Note that the difference operator applied to the transformed data removes completely trend and seasonality).

The results from fitting linear and quadratic trend lines to the deseasonalised actual and transformed data and their plotts are shown in Figures 5 and 6.

The results from fitting model E are shown in Figures 7, 8, 9 and 10. First we fit model E1, then we analyse the residuals and identify a tentative model for them and finaly we reestimate the whole model after excluding any non-significant parameters. The resulting is model E3. We observe that the seasonality of the series is of the multiplicative type except for March whose seasonality is additive. In this respect model E differs from model D. We note also the negative signs of the coefficients of months February, November and December.

FITTING ACCURACY

Some measures of fitting accuracy are given in Table 1. Direct comparisons are not possible between models fitted to the original data and those fitted to the transformed. Generally the fitting accuracy increases when we introduce a non-linear trend and/or a Sarima model is considered.

We note the following:

The SSE of the H-W log add. model is less than the SSE of the B-J model taking into account the number of obs. and the pattern of the residuals.

The fitting accuracy of model A increases when we correct for 1st order autocorrelation in the residuals and becomes even better when we include a SARIMA model for the residuals (in this case its SSE is less than those of the H-W and B-J). The same holds for models B,C,D and E.

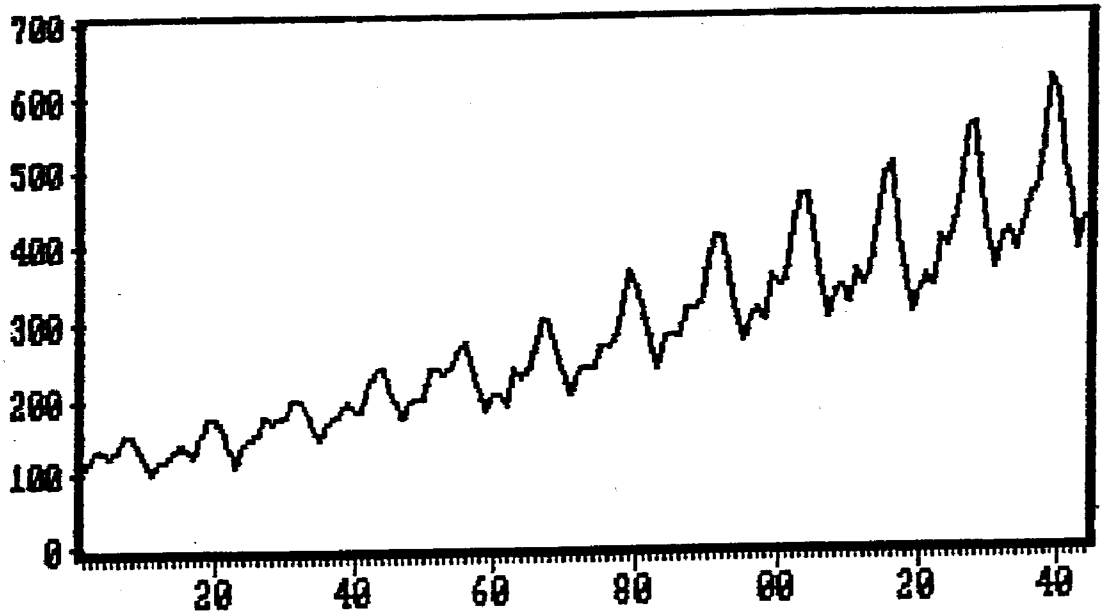

Figure 1. The Airline data.

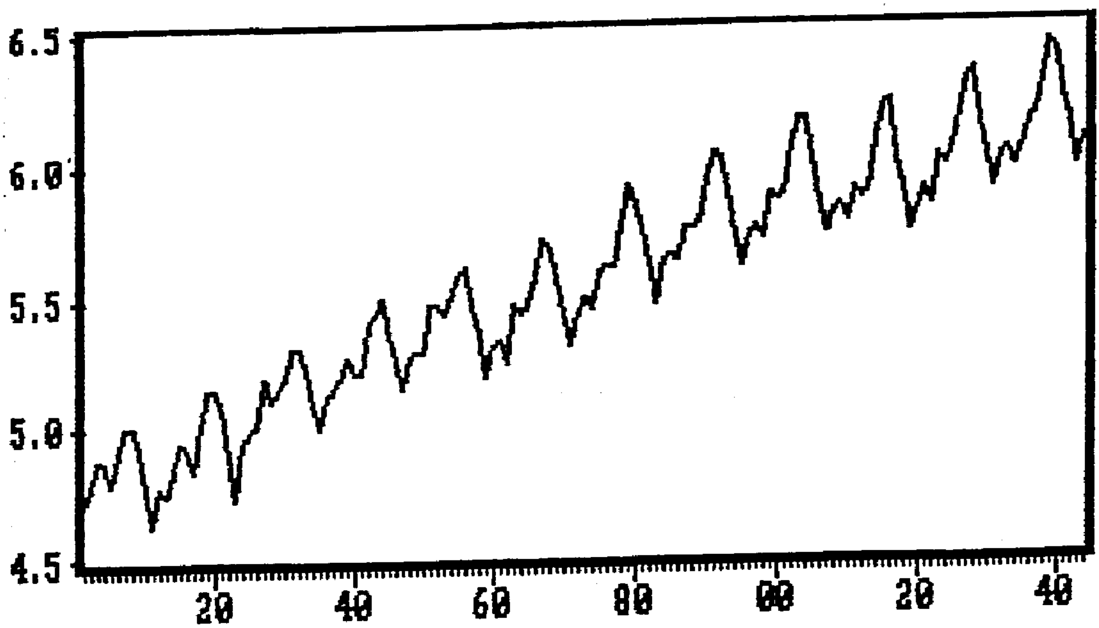

Figure 2. Logarithms of the Airline data.

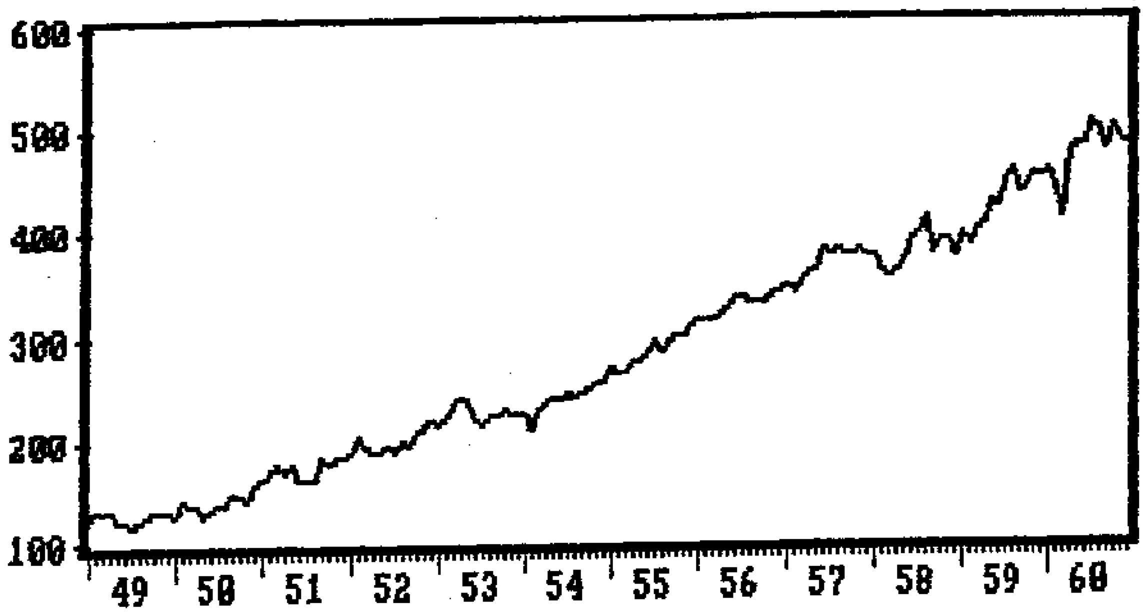

Figure 3. Deseasonalised data.

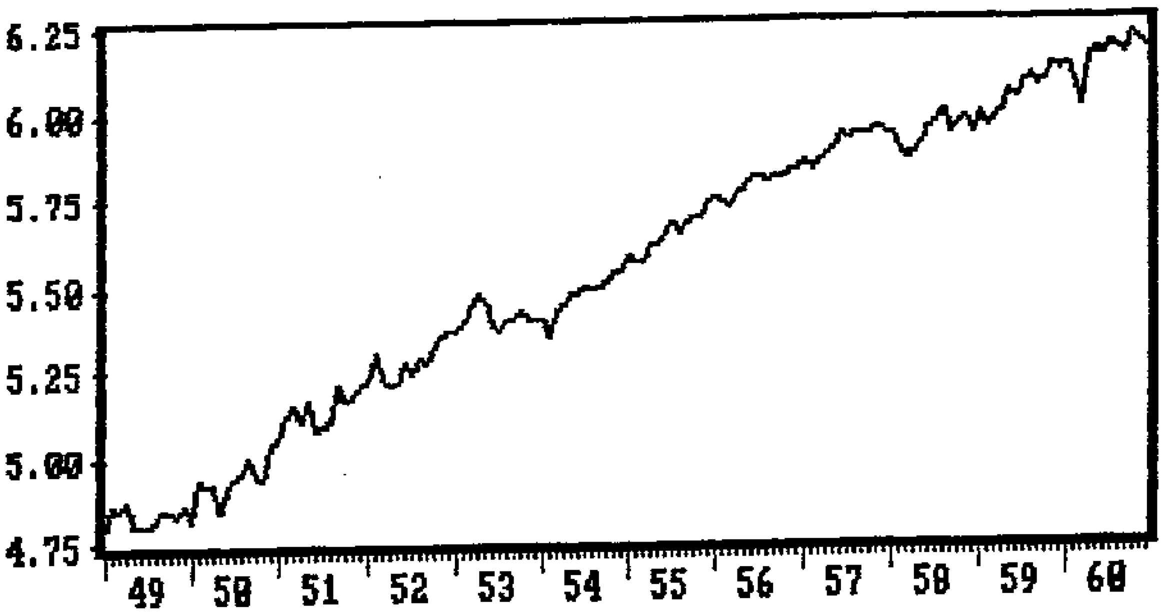

Figure 4. Deseasonalised transformed data.

SMPL 1949.01 - 1960.12
144 Observations
LS // Dependent Variable is YA

Variable	Coefficient	Std. Error	T-Stat.	2-Tail Sig.
C	88.437341	2.8430368	31.106646	0.000
T	2.6473202	0.0340193	77.818220	0.000
R-squared	0.977088		Mean of dependent var	280.3681
Adjusted R-squared	0.976927		S.D. of dependent var	111.7157
S.E. of regression	16.96945		Sum of squared resid	40890.65
Durbin-Watson stat	0.417785		F-statistic	6055.675
Log likelihood	-611.0439			

SMPL 1949.01 - 1960.12
144 Observations
LS // Dependent Variable is YA

Variable	Coefficient	Std. Error	T-Stat.	2-Tail Sig.
C	113.58916	3.2669088	34.769614	0.000
T	1.6136840	0.1040190	15.513362	0.000
T2	0.0071285	0.0006949	10.258417	0.000
R-squared	0.986880		Mean of dependent var	280.3681
Adjusted R-squared	0.986694		S.D. of dependent var	111.7157
S.E. of regression	12.88656		Sum of squared resid	23414.94
Durbin-Watson stat	0.727753		F-statistic	5303.040
Log likelihood	-570.9019			

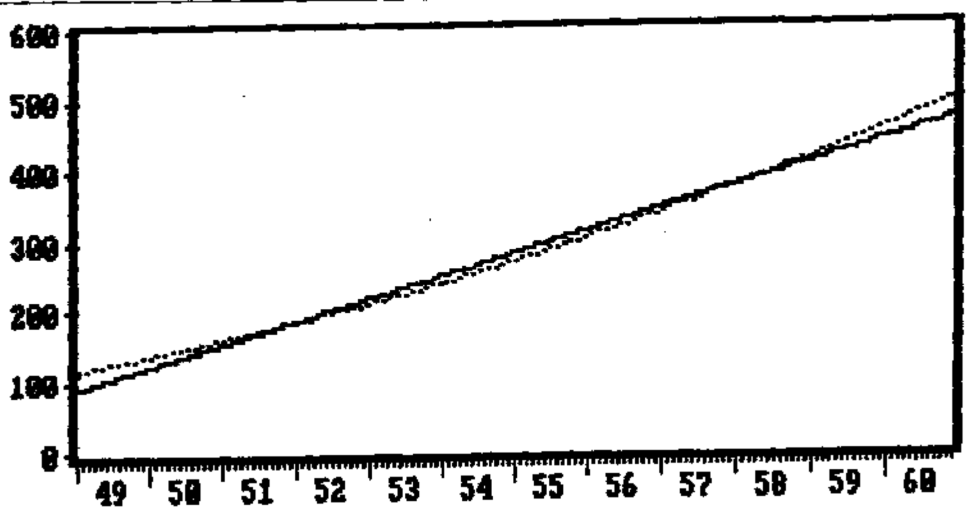

Figure 5. Estimated linear and quadratic trend lines from the deseasonalised data.

SMPL 1949.01 - 1960.12
144 Observations
LS // Dependent Variable is LA

Variable	Coefficient	Std. Error	T-Stat.	2-Tail Sig.
C	4.8136040	0.0093354	515.62782	0.000
T	0.0100543	0.0001117	90.007108	0.000

R-squared	0.982774	Mean of dependent var	5.542544
Adjusted R-squared	0.982653	S.D. of dependent var	0.423059
S.E. of regression	0.055721	Sum of squared resid	0.440887
Durbin-Watson stat	0.474180	F-statistic	8101.280
Log likelihood	212.4650		

SMPL 1949.01 - 1960.12
144 Observations
LS // Dependent Variable is LA

Variable	Coefficient	Std. Error	T-Stat.	2-Tail Sig.
C	4.7379994	0.0113572	417.18063	0.000
T	0.0131614	0.0003616	36.396100	0.000
T2	-2.143D-05	2.416D-06	-8.8700270	0.000

R-squared	0.988943	Mean of dependent var	5.542544
Adjusted R-squared	0.988787	S.D. of dependent var	0.423059
S.E. of regression	0.044799	Sum of squared resid	0.282983
Durbin-Watson stat	0.738016	F-statistic	6305.775
Log likelihood	244.3899		

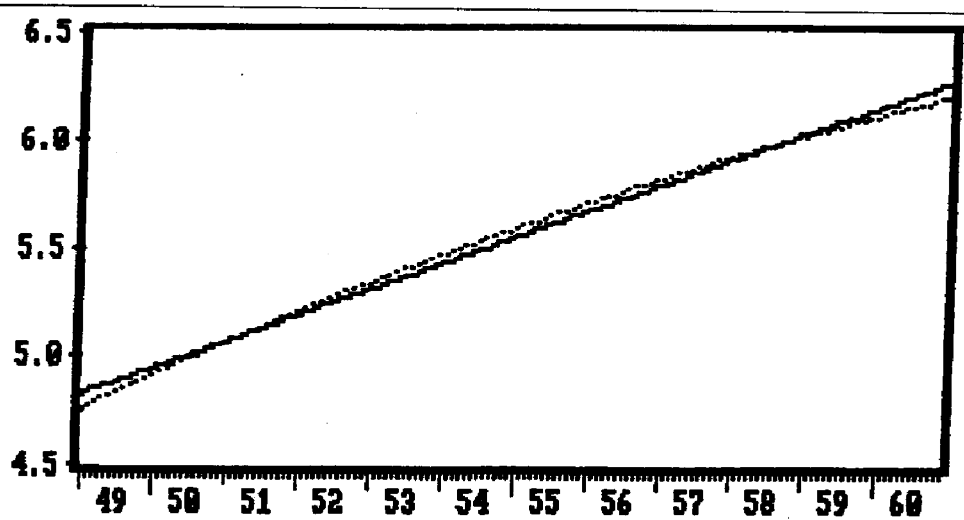

Figure 6. Estimated linear and quadratic trend lines from the tranformed and deseasonalised data.

SMPL 1-144
144 Observations
LS // Dependent Variable is Y

Variable	*Coefficient*	*Std. Error*	*T-Stat.*	*2-Tail Sig.*
C	105.59794	6.4427013	16.390320	0.000
T	1.3978831	0.1145795	12.200116	0.000
T2	0.0068483	0.0006206	11.034512	0.000
DA	10.305352	8.8282935	1.1673097	0.246
DB	26.191126	8.8775735	2.9502573	0.004
DC	11.819035	8.9281225	1.3237984	0.188
DD	2.4862811	8.9799548	0.2768701	0.782
DE	8.0704875	9.0330845	0.8934365	0.374
DF	7.9166424	9.0875251	0.8711549	0.386
DG	6.9093610	9.1432899	0.7556756	0.452
DH	10.323993	9.2003917	1.1221254	0.264
DI	-4.2441816	9.2588429	-0.4583922	0.648
DJ	-12.136946	9.3186557	-1.3024353	0.196
DK	1.8307221	9.3798416	0.1951762	0.846
M2	-0.2849671	0.1114715	-2.5564121	0.012
M3	-0.0352605	0.1114922	-0.3162594	0.752
M4	0.0929426	0.1115268	0.8333663	0.407
M5	0.2566935	0.1115751	2.3006346	0.023
M6	0.6957941	0.1116372	6.2326351	0.000
M7	1.1989972	0.1117131	10.732825	0.000
M8	1.1605336	0.1118027	10.380192	0.000
M9	0.4183987	0.1119060	3.7388402	0.000
M10	0.1010890	0.1120230	0.9023953	0.369
M11	-0.2677941	0.1121536	-2.3877450	0.019
M12	-0.1031691	0.1122977	-0.9187106	0.360

R-squared	0.992603	Mean of dependent var	280.2986
Adjusted R-squared	0.991112	S.D. of dependent var	119.9663
S.E. of regression	11.31022	Sum of squared resid	15222.61
Durbin-Watson stat	0.696528	F-statistic	665.3910
Log likelihood	-539.8993		

Figure 7. The estimated model E1.

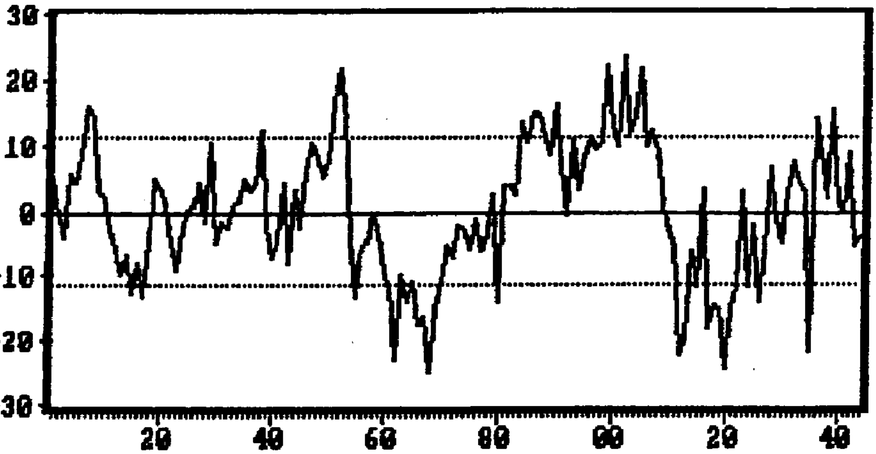

Figure 8. Plot of residuals after fitting model E1.

SMPL 1-144
144 Observations
IDENT R

Autocorrelations	Partial Autocorrelations		ac	pac
*******	*******	1	0.6504	0.6504
******	***	2	0.5799	0.2719
*****	*	3	0.5058	0.1018
****	*	4	0.3645	-0.1078
****	*	5	0.3806	0.1377
***		6	0.3363	0.0485
**	**	7	0.2054	-0.1701
**		8	0.1740	-0.0431
**	*	9	0.1656	0.1088
	***	10	-0.0109	-0.2638
		11	0.0069	-0.0055
		12	-0.0348	0.0418
*		13	-0.0875	0.0008
*	*	14	-0.0771	-0.0860
**	*	15	-0.1545	-0.0812
**	*	16	-0.2428	-0.0970
**		17	-0.2163	0.0105
**		18	-0.2292	-0.0051
***	**	19	-0.3160	-0.1516
***		20	-0.2797	-0.0332
***	*	21	-0.2872	0.0802
***	*	22	-0.3342	-0.1250
**	*	23	-0.2333	0.0683
***	*	24	-0.3328	-0.1492

S.E. of Correlations 8.333334E-02 Q-Stat. (24 lags) 322.6171

Figure 9. Identification of the residuals after fitting model E1.

SMPL 3 -144
142 Observations
LS // Dependent Variable is Y
Convergence achieved after 5 iterations

Variable	*Coefficient*	*Std. Error*	*T-Stat.*	*2-Tail Sig.*
C	108.35616	8.8618192	12.227304	0.000
T	1.3923800	0.2647221	5.2597798	0.000
T2	0.0066243	0.0016732	3.9589494	0.000
M2	-0.1796338	0.0340142	-5.2811401	0.000
DB	21.499364	2.7938317	7.6952970	0.000
M4	0.2184023	0.0398932	5.4746721	0.000
M5	0.2835708	0.0418228	6.7802926	0.000
M6	0.7791687	0.0437225	17.820769	0.000
M7	1.2853358	0.0444450	28.919682	0.000
M8	1.2308310	0.0440505	27.941346	0.000
M9	0.5228840	0.0428880	12.191848	0.000
M10	0.0644680	0.0413346	1.5596603	0.119
M11	-0.3863747	0.0372615	-10.369261	0.000
M12	-0.0829204	0.0348498	-2.3793677	0.017
MA(12)	0.2139384	0.1019261	2.0989570	0.036
AR(1)	0.4441844	0.0883178	5.0293894	0.000
AR(2)	0.2656487	0.0890822	2.9820634	0.003

R-squared	0.995336	Mean of dependent var	282.6268
Adjusted R-squared	0.994739	S.D. of dependent var	119.1759
S.E. of regression	8.644170	Sum of squared resid	9340.210
Durbin-Watson stat	1.948918	F-statistic	1667.245
Log likelihood	-498.7135		

Figure 10. The estimated model E3.

TABLE 1

	R^2	$\overline{R^2}$	S.E. of regr.	D-W	SSE	Q(24)
B-J (131 obs)					0,1736	23,8
H-W log add					0,1860	44,2
A1	0,9835	0,9820	0,0593	0,425	0,4607	396,4
2	0,9935	0,9929	0,0369	2,192	0,1759	31,0
3	0,9940	0,9933	0,0356	2,005	0,1600	26,5
B1	0,9892	0,9881	0,0482	0,648	0,3020	207,0
2	0,9940	0,9934	0,0356	2,093	0,1625	31,0
3	0,9944	0,9937	0,0345	2,015	0,1485	27,0
H-W mult.					16.429,9	62,7
C1	0,9829	0,9813	16,4017	0,404	35.240,9	419,9
2	0,9938	0,9931	9,9076	2,380	12.662,7	77,8
3	0,9945	0,9939	9,3089	2,022	10.918,6	41,0
D1	0,9912	0,9903	11,8063	0,778	18.120,4	231,1
2	0,9944	0,9938	9,4206	2,184	11.359,8	69,8
3	0,9948	0,9941	9,1308	1,964	10.421,5	42,7
E1	0,9919	0,9911	11,3434	0,777	16.727,6	264,7
2	0,9949	0,9943	9,0116	2,303	10.394,7	55,5
3	0,9953	0,9947	8,6442	1,949	9.340,2	32,6

According to our data model B fits better than model A due to the quadratic trend which is included. In this case the SARIMA model is obviously less important.

The SSE of the H-W mult. model is less than the SSE of model E1.

Model D also fits better the data than model C while model E fits even better for the reasons discussed above.

FORECASTING ACCURACY

Some measures for ex-post forecasting accuracy for 12, 24 and 36 time horizons ahead (i.e. using the first 132, 120 and 108 obs.) are given in Tables 2, 3 and 4. Note that the negative sign in the mean error means that forecasts are overestimates of actual obs and in this case MAPE is more suitable for comparing forecasts of different time length.

We observe that:

a) The forecasting accuracy of the two Holt-Winters models is not the same and in some cases is better than the Box-Jenkins.

b) Forecasts from model B are more accurate from those of model A.

c) Forecasts from model D are more accurate from those of model C in the two previous cases.

d) Forecasts from model E are more accurate from all the others in the two first cases.

12 time period ahead forecasts

In this case the H-W log add, the B-J and the A models give over-estimated forecasts. We suspect that this is due to the antilog transformation used.

According to their accuracy the models are ranked as follows:

E1 E2 E3 D2 D3 C3 B-J H-W B C1 A

Considerable improvement from including an ARMA term is observed in models A and C.

TABLE 2

	RMSE	MAE	ME	MAPE	Theil's			
					U_2	U_m	U_r	U_d
H-W mult.	25,2	18,51	4,6	3,8	0,052	0,033	0,21	0,757
H-W log add.	22,6	15,36	-8,76	3,4	0,047	0,15	0,164	0,686
B.-J.	17,9	12,55	-10,8	2,76	0,037	0,365	0,0009	0,634
A1	40,2	35,05	-35,05	7,84	0,083	0,762	0,02	0,217
2	31,5	25,19	-24,64	5,66	0,065	0,613	0,02	0,367
3	30,2	24,2	-24,09	5,43	0,063	0,635	0,015	0,35
B1	24,0	18,48	6,74	3,78	0,05	0,079	0,24	0,681
2	24,7	19,11	7,63	3,9	0,0511	0,096	0,237	0,667
3	23,1	17,93	7,64	3,65	0,048	0,109	0,234	0,656
C1	32,8	31,7	29,1	6,7	0,0680	0,786	0,0018	0,212
2	24,0	21,4	15,5	4,5	0,0500	0,416	0,0132	0,571
3	16,4	10,4	1,3	2,3	0,0340	0,007	0,0318	0,9616
D1	15,3	10,9	3,8	2,4	0,032	0,063	0,0065	0,931
2	15,3	9,7	0,4	2,2	0,032	0,0006	0,0186	0,981
3	15,7	10,4	-4,6	2,3	0,033	0,0858	0,0248	0,8894
E1	11,7	8,8	4,2	1,9	0,0242	0,1314	0,0226	0,846
2	12,3	9,2	2,9	2,0	0,0256	0,0551	0,0549	0,89
3	12,8	9,9	-3,7	2,2	0,0265	0,0852	0,0544	0,86

TABLE 3

	RMSE	MAE	ME	MAPE	Theil's U_2	U_m	U_r	U_d
H-W mult.	51,9	44,8	43,6	9,4	0,1133	0,7047	0,1327	0,1626
H-W log add.	28,5	23,3	19,1	4,9	0,0623	0,4458	0,1569	0,3973
B.-J.	37,2	33,5	33,3	7,2	0,0811	0,8	0,0768	0,1229
A1	46,9	43,0	-43,0	9,9	0,1023	0,841	0,0014	0,158
2	33,5	27,2	-25,8	6,2	0,073	0,5962	0,0076	0,396
3	26,0	19,2	-15,7	4,3	0,057	0,364	0,012	0,624
B1	21,7	15,8	-0,8	3,5	0,047	0,0013	0,305	0,694
2	27,0	22,4	16,5	4,7	0,059	0,3723	0,227	0,401
3	35,5	30,0	27,6	6,2	0,077	0,6058	0,1678	0,2264
C1	31,5	27,6	26,5	6,0	0,069	0,709	0,008	0,286
2	31,9	28,8	27,8	6,4	0,07	0,76	0,0054	0,2345
3	35,1	32,3	31,9	7,2	0,077	0,826	0,003	0,1713
D1	14,2	10,6	-3,1	2,5	0,031	0,047	0,013	0,94
2	14,1	11,0	6,5	2,5	0,031	0,0212	0,0002	0,788
3	26,2	24,2	22,9	5,4	0,057	0,0763	0,0009	0,236
E1	11,8	9,2	-2,5	2,1	0,026	0,047	0,027	0,927
2	13,5	11,1	9,2	2,5	0,030	0,469	0,006	0,526
3	30,1	27,8	27,8	6,2	0,066	0,853	0,004	0,143

TABLE 4

	RMSE	MAE	ME	MAPE	Theil's			
					U_2	U_m	U_r	U_d
H-W mult.	28,7	24,3	3,5	5,6	0,06595	0,015	0,4006	0,5843
H-W log add.	45,3	40,2	-39,7	9,8	0,1041	0,7679	0,0022	0,2299
B.-J.	36,2	32,8	-32,6	8,0	0,0830	0,8120	0,0000	0,1880
A1	63,3	59,2	-59,2	14,2	0,1454	0,8728	0,0078	0,1194
2	60,8	56,1	-56,1	13,4	0,1395	0,851	0,0137	0,1354
3	62,4	57,7	-57,7	13,8	0,1432	0,8572	0,0176	0,1252
B1	46,2	41,8	-41,7	10,3	0,106	0,8154	0,0013	0,1834
2	36,4	31,4	-30,8	7,8	0,0836	0,7151	0,0150	0,2700
3	29,4	24,6	-22,4	6,2	0,0676	0,5797	0,0768	0,3435
C1	27,2	22,0	17,8	4,9	0,0625	0,4273	0,0585	0,5142
2	23,8	19,8	9,7	4,5	0,0546	0,1673	0,0906	0,7421
3	21,3	18,9	2,4	4,5	0,0489	0,0125	0,0984	0,8891
D1	38,2	35,6	-35,6	8,6	0,0878	0,8668	0,0001	0,133
2	37,7	34,9	-34,9	8,4	0,0866	0,8572	0,0008	0,1419
3	37,4	34,8	-34,8	8,4	0,0858	0,8649	0,0012	0,1339
E1	38,1	35,9	-35,9	8,7	0,0874	0,8912	0,0002	0,1087
2	34,2	32,2	-32,2	7,8	0,0785	0,8853	0,0000	0,1147
3	33,5	31,6	-31,6	7,7	0,0769	0,8916	0,0000	0,1084

24 time period ahead forecasts

In this case the ranking order is the following:

E1 E2 D1 D2 B1 B2 A3 H-W log add B-J H-W mult A1

We observe that the inclusion of the ARMA term decreases the forecasting accuracy. Also model A gives overastimated forecasts.

36 time period ahead forecasts

In this case the ranking order is the following:

C3 C2 C1 H-W mult B3 E3 B-J D H-W log add B1 A

In this case all models except C and H-W mult give overestimated forecasts. We suspect that the reason is the ommision of the 36 obs, in which case the extrapolation of the linear trend gives better results.

Finally, plots of the actual data and 12, 24 and 36 time ahead forecasts from model E and 36 time ahead forecasts from model C are shown in Figures 11, 12, 13 and 14.

CONCLUSIONS

We conclude that the successful application of seasonal interaction models for forecasting, depends on their specification. A similar process to the one suggested by Box and Jenkins must be followed in this case also.

From the empirical findings of this investigation and our experience after using these models in some other cases, we propose the direct use of the multiplicative or mixed versions instead of the log-additive, when the seasonality is not obviously additive. The type of trend and the SARIMA model fitted to the residuals are also very important factors for this type of models.

Finally, the obvious advantages of these models make them worth a trial.

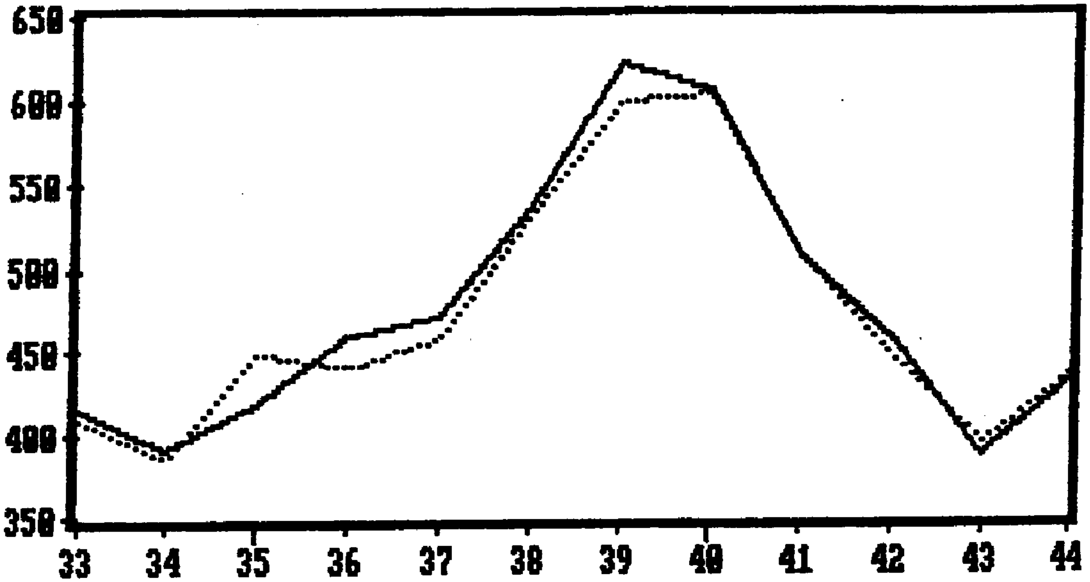

Figure 11. Actual data and forecasts from model E1.

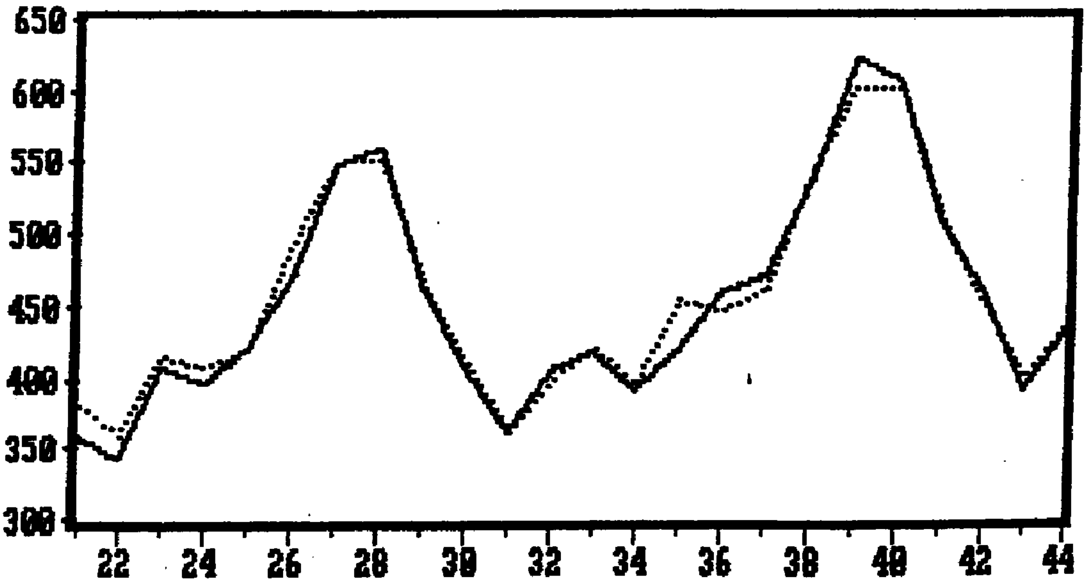

Figure 12. Actual data and forecasts from model E1.

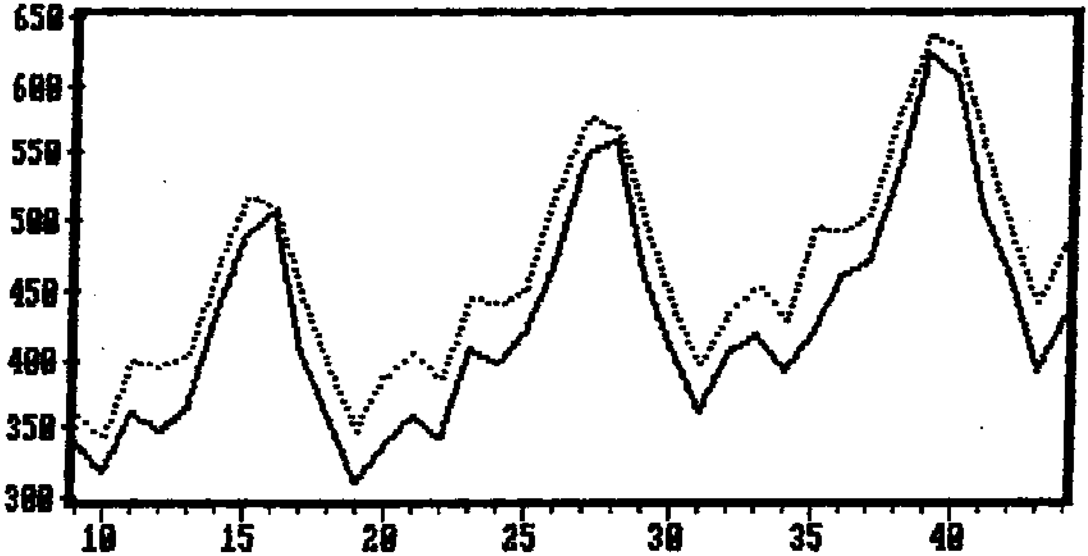

Figure 13. Actual data and forecasts from model E1.

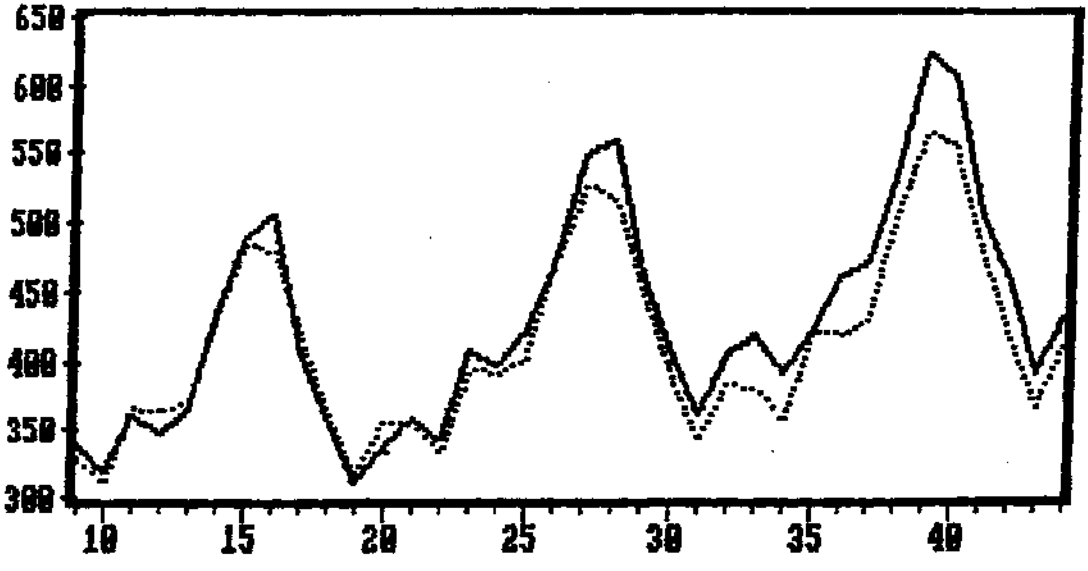

Figure 14. Actual data and forecasts from model C3.

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