ENDOGENISING TOTAL FACTOR PRODUCTIVITY: 
THE FOREIGN DIRECT INVESTMENT CHANNEL 
IN THE CASE OF BULGARIA (2004-2013)

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Abstract
This paper estimates the contribution of Foreign Direct Investment (FDI) to the Total Factor Productivity (TFP) of Bulgaria for the period 2004-2013. As predicted by theory, a positive relationship between TFP and FDI is documented. The standard Ramsey (optimal) growth model, augmented through the FDI channel is used to compare the rate of convergence to an identical setup without FDI. Convergence simulations prove that ignoring the implications of this model leads to a distorted view of the growth path of the economy. The results of the study can serve as justification for developing governmental strategies to attract FDI inflows.

\textbf{JEL Classification:} E13, E17, E22, O33
\textbf{Key Words:} Simulation, Endogenous Growth, TFP, FDI

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1. Introduction

Foreign direct investment (FDI) is an inevitable consequence of an open-market economy and its effect on economic growth has been one of the most noticeable and discussed topics in the past several decades. FDI is a bridge between economies and it is considered a tool for transferring skills, technology, and knowledge between countries. The impact of FDI is expected to be growth-enhancing through the introduction and incorporation of new inputs of technologies, which influence both labour and physical capital efficiency. Some of the documented effects of FDI are unemployment reduction, improved population welfare, productivity growth (TFP) and accelerated economic growth.

A number of studies associate increase in Total Factor Productivity (TFP) with increase in FDI; however, an unconditionally positive relationship between FDI and TFP has not yet been proven. Some studies provide proof that the impact of FDI is indeed positive, but it seems that results depend on the level of development and openness of the economy. Because FDI is seen as a key channel for transferring more advanced organisational forms and technologies in industrialised and developing countries (Isaksson, 2007), evidence documenting the positive impact of FDI on TFP would provide justification for the introduction of policies and the development of governmental strategies to attract FDI inflows.

Bulgaria is a good case for exploring this subject as the country is a transitional economy. Based on the level of country development, it can be supported that the country needs to find ways of accumulating capital and knowledge. Proving that FDI is a channel satisfying these needs will encourage expansion in this direction. Furthermore, Bulgarian studies and empirical experiments are scarce, so an important objective of this paper is to provide a theoretical alternative for both policymakers and future academic researchers.

This research provides a brief review of existing literature on FDI, TFP and knowledge accumulation and applies the theory for the Bulgarian economy, proving the positive link between the variables. Furthermore, the measured impact is incorporated in simulations predicting future development of the economy. The remainder of the paper is organised as follows: in Section 2, we will look at the existing literature on the connection between FDI and TFP growth. We will include a brief analysis of the results of the studies and comment on the differences between them. Section 3 describes the theoretical framework and the structure of the model and Section 4 describes the estimation strategy and data. Section 5 presents data analysis, econometric results and concerns. Section 6 will be used for the formulation of predictions based on Section 5 results and Section 7 is reserved for conclusions.
2. Literature Review

We will focus on articles and studies that describe the relationship between TFP and FDI. Even though this connection can be studied at a micro level or spill-over effects of FDI within a certain sector, we will focus on the aggregate level. Few studies so far have examined the impact of FDI on TFP at the macro level with predominantly positive results; however, there are several authors who argue that variables might be negatively related. We will discuss both of these options and proceed to examine the Bulgarian case.

TFP has long been perceived as an exogenous variable that in the standard output model. However, this is not observed in open market economies, as stated by Romer (1990). In his study of TFP endogeneity, he finds that integration can increase growth, since integration into world markets means openness and possibility to invest and receive investments from abroad. An inference that can be made from this finding is that FDI, as the channel of moving funds between economies, would also lead to increased productivity growth, similarly to efficiency improvement due to the presence of trade.

Arisoy (2012) takes a look at the effect of FDI on both TFP and economic growth for Turkey for the period 1960–2005. His empirical results, based on regressing TFP and GDP on FDI only, show that FDI has a positive impact on both, through technological spill-over and capital accumulation. Pessoa (2005) receives positive results for a panel of OECD countries and concludes that FDI has a positive impact on a host country’s TFP. He attributes this to the fact that FDI is a channel through which technologies are transferred internationally. In addition, Woo (2009) shows that for the period 1970-2000, in a large sample of countries, FDI had positive effect on TFP growth.

Positive linkages between TFP and FDI vary in nature, particularly for developing and transitional economies. For example, Zhang (2002) studies the contribution of FDI to productivity growth in cross-region analysis in China for the 1984 to 1997 period, and finds a bidirectional causal linkage between FDI and TFP. The results of the study suggest that China’s growth is largely due to rapid expansion of physical investment in fixed assets from FDI and not so much to technology transfer. This is a result of inefficiency and lack of capability to assimilate technology. Nevertheless, FDI invested in more labour-intensive sectors did have positive effect on labour productivity.

Even when a positive link is shown between variables, developed countries seem to experience the effects of FDI in a different manner than developing ones. In Keller and Yeaple’s (2003) study of plants in the US (1987-1996), the FDI effect was more pronounced in more technologically oriented sectors because of better communication with international companies. More than 10% of the increase in productivity growth is attributed to FDI spill-over effects. FDI effect seems to get more concrete the more organised and advanced an economy is.
Likewise, the positive relationship authors perceive can be country specific. Mello (1999) estimates the impact of FDI on capital accumulation, output and TFP growth and comes to the conclusion that FDI influence is country-specific due to factors that are unobservable in time series analysis. The impact of FDI depends on whether the receiving country is a leader or a laggard, since effects of technological transfer are lower in a country that is developing. These observations are based on a time-series panel data for a sample of OECD and non-OECD countries in the period 1970-1990.

The absence of direct positive effect of FDI on TFP is usually explained on the grounds of low absorption efficiency of the economy, thus making it impossible for the country to benefit from any increase in human capital and technology (Borensztein et al., 1998). Furthermore, the levels of economic freedom, openness of the economy and establishment of efficient financial environment also play an important role. For example, a negative relationship between FDI and TFP was present in a study by Sadik and Bolbol (2001). For several developing Arab countries (Egypt, Jordan, Morocco, Oman, Saudi Arabia and Tunisia), they investigate whether FDI affects TFP through technology spill-over effects to find that FDI actually had a “very significant and negative effect” on most countries included in the study. However, these authors clearly establish that the results might be caused by inefficient governmental policies and institutions, lack of investment efficiency and inadequate appreciation and availability of technological innovation.

Given the inconsistencies in literature, a model is being proposed based on the idea that FDI has an effect on TFP; however, additional variables that could influence TFP will not be excluded. So far, most studies have incorporated additional variables expected to have positive influence on TFP; nevertheless, we have decided to include aspects of the economy that could also have negative effect on TFP.

3. Model Setup

In neoclassical growth models (Solow, 1956), technological progress is modelled as being determined outside of the model, in the absence of exogenously growing TFP, while growth monotonically decreases and asymptotically goes to zero, as the economy converges to the steady state. Modern growth theory (Romer, 1990) tries to explain how progress arises and, therefore, can be enhanced - in other words, it tries to endogenise the variable. For that purpose, the neoclassical model is expanded to incorporate explanations for knowledge generation and accumulation.

Bulgaria is a perfect case for applying and studying the modern view over technological change, as the country had to go through a process of knowledge and capital accumulation in order to come out of the crisis in the 1990s. After the disbandment of the communist block and the dissolution of the “iron curtain”, Bulgaria faced the challenge of acquiring, developing and accumulating modern
knowledge and capital. Like many post-communist countries, a combination of economic failures, lack of understanding of market economy principles and selfish practices of political leaders led the country to hyperinflation in 1997. Economic conditions did not start significantly improving until the early 2000s. The country was forced to open its market to western influence and FDI was one of the channels to achieve this.

FDI was expected to expand the productivity of the country through labour force training, skill acquisition, and introduction of alternative management practices and organisational arrangements. These were expected to be implemented through cooperation with foreign companies and acceptance of foreign investment, which means that we expected the growing FDI into the country to have a positive effect on productivity. This effect has been proven for some industrialised countries that have better data, but it needs to be further proven for developing countries, such as Bulgaria. The country needs to promote innovation and progress, and demonstrating that FDI is, in fact, a tool for achieving this goal, could further promote practices enhancing international cooperation.

Because most theories suggest that FDI has a spill-over effect on technological change, we will take TFP as a dependent variable indicating technological progress. TFP does not only reflect technological improvement, but also increase in a country’s knowledge and efficiency. The idea of learning-by-doing and its economic implications was developed and expanded by Arrow (1962), who incorporated the notion that knowledge changes lead to shifts in the production function. In the model, every new machine or any capital accumulated is capable of changing the environment while learning to use this capital is taking place. This model, however, is oversimplified as it does not include additional variables that influence the learning process. Nevertheless, we will base our assumptions and model on the idea that learning reflects increase in TFP, and occurs as a side effect of the production of new capital.

The model used in calculating the influence of FDI and proving the assumptions presented above on the effect of FDI on TFP is based on aforementioned idea of knowledge accumulation through learning-by-doing. In this model an increase of TFP or increase of knowledge, is a function of the increase in capital. Similarly it will here be assumed that TFP is a function of FDI:

\[ A_t = B_t F_t^\gamma e^{zt} A_t = B_t F_t^\gamma e^{zt} \]  

or

\[ \ln A_t = \ln B_t + \gamma \ln F_t + \varepsilon_t, \ln A_t = \ln B_t + \gamma \ln F_t + \varepsilon_t, \]
where $A$ is TFP in stationary form (as discussed in section 5), $B$ is a shifting parameter representing additional variables influencing TFP, $F$ is FDI stock and $\gamma$ is a parameter between 0 and 1 (based on the natural phenomenon of diminishing returns to rival production factors). An important point that needs to be added is that for the purposes of this model FDI considered an exogenous variable. We have regarded FDI as exogenous, simply because Bulgaria is a small open economy that does not leave an important imprint on the world’s economy. While it is true that FDI’s endogeneity is an important issue that needs further exploration, research into it is not the primary purpose of this study.

What we have decided to employ for the purpose of this study is quarterly data of FDI stock in millions of Bulgarian currency in real terms (2010 prices). We preferred stock values due to the existing delay in the effect of any investment on production due to the time needed to build physical capital, teach workers to use new equipment or incorporate a new organisation structure. As FDI flows can rarely be incorporated into the existing system at the time they have been received, we consider stock to be a better measure of FDI impact on TFP (Arisoy, 2012). In the model of this paper, and due to data limitations (no distinction can be made between investment vs. non-investment), we assume all FDI is structural. Thus, our results are to be taken as an upper bound effect on TFP.

The effect of FDI is represented by $\gamma$ in the model presented and we expect to find it to be positive, as we expect it to have enhancing properties. The shifting parameter is included in the model, as there are a number of variables that could enhance or decrease the influence of FDI. In the econometric analysis, the regression for this equation would take the following form:

$$ln\bar{A}_t = a_0 + b_1 lnF_t + b_2 X_t + b_3 lnY_t + b_4 lnZ_t + \epsilon_t$$

where $b_1 = \gamma$, $F$ is FDI stock and $X$, $Y$, $Z$ are control variables that lie in $B$ and impact the effects of FDI on TFP. All variables are de-trended following the methods in Section 5.

The additional variables we have decided to include into our regression are Government spending on Health, Education and Social protection and spending for Research and Development (R&D). We have decided to incorporate these variables because of their probable effects on the productivity of the country. Government spending on health, education and R&D are straightforward and are expected to have positive effect on TFP, since they are intended to make the labour force more productive.

1. Due to its small size, government spending on R&D always needs to be used cautiously in regressions. Nevertheless, it varies sufficiently for the figure to be individually significant in the regression. In addition, the F-test for joint significance cannot reject the joint importance of R&D and the remaining right-hand-side variables. Dropping R&D could bias the FDI coefficient estimated downward.
Expenditure for social protection is expected to have a negative effect on TFP, since it provides an excuse for people to be absent from work, thus decreasing productivity. Such expenditures cover sickness / healthcare benefits (paid sick leave, medical care and the provision of medication), disability benefits, old-age benefits, survivors’ benefits, family and children benefits (pregnancy, childbirth, childbearing and caring for dependent family members), unemployment benefits, housing benefits and others. Unemployment benefits are a relatively small part of expenditure on social protection. A much larger share represents social pensions, widow’s benefits, invalidity pensions, children benefits, in-kind benefits (energy subsidies, timber, electricity vouchers, food stamps, food packages, etc.). For example a handicapped person’s pension is for life, and it is much more generous than the unemployment payout, which makes it more attractive for non-workers and could lead to some embezzlement schemes.

4. Data

Measuring TFP could become problematic if incorrect data and methods are used. Therefore, we are going to replicate the method already established by Ganev (2005) for measuring Bulgarian TFP. The period he covered was 1991-2003, using annual data; however, in this study, we are going to examine quarterly data from the period 2004-2013, which was selected because its start corresponds with the increase in structural FDI in Bulgaria. Given that data is quarterly, we decided to take an earlier end point, in order to avoid problems with later data revisions.

As we have established, Total Factor Productivity represents technological change and productivity. It also represents an additional factor that influences GDP growth regardless of relative change in capital and labour. In this study, TFP is calculated using the Cobb-Douglas production function:

\[ Y_t = A_t K_t^\alpha L_t^\beta \]  

In equation (4), \( Y_t \) represents real GDP for a specific time \( t \), \( A_t \) is TFP, and \( L_t \) and \( K_t \) are labour and capital, respectively. The symbols \( \alpha \) and \( \beta \) represent output elasticity of capital and labour, respectively, and \( \alpha + \beta = 1 \), since we assume constant return to scale. \( A_t \) is current year development level and is found as a residual from the equation, \( L_t \) is measured as the total number of hours worked during the current year and \( K_t \) is the real value of physical capital in the current year.

Data on labour and GDP are collected by the National Statistical Institute in Bulgaria. Capital is calculated using the perpetual inventory method (Appendix 1) and \( \alpha \) is found by calculating the ratio of compensation of employees and net mixed income to GDP. All data are seasonally adjusted and in real terms (2010 prices) – the process of adjustment can be found in Appendix 2. Data on FDI stock are collected by the Bulgarian National Bank and represents only the stock of inward FDI for the period under study.
Values for Health, Education, Social welfare, and R&D expenditure are calculated using Eurostat data. In the original dataset, the values of variables were presented as annual GDP percentage rates; however, as we need quarterly information (unfortunately not available for either of the variables), we have calculated a time series for each variable using the quarterly GDP variable. Because GDP is, in fact, in millions of Bulgarian currency, seasonally adjusted and in real terms, the 4 variables are also presented in the same manner.

When creating a scatter plot of TFP and FDI stock values in Bulgaria for the period 2004-2013 (TFP values on the vertical axis and FDI values on the horizontal one) a negative relationship for the first two years is can be observed. For the next 8 years a boom in TFP growth is present, although FDI barely increased. A reason for this might be possibly delayed FDI effects on TFP. In fact, if we incorporate the 8th lag of FDI stock in the same scatter plot (equal to a two-year gap), we receive the graph in Figure 1, which supports the claims of a positive relationship between variables.

Figure 1. TFP-FDI scatter plot

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3. Data source - BNB (2014), authors’ calculations.
The decision to use the 8th lag is further reinforced by the idea that any investment needs time to produce results. There are several studies on the time-to-build and time-to-plan theories, the most prominent one being that by Kydland and Prescott (1982). Although we are aware that they were not the first to make this observation, we chose their reference because these authors were the first to operationalise the mechanism in a general dynamic equilibrium context. These authors found that there was no evidence that capital goods could be built faster if more money was invested, which means that the time needed for building an investment is independent of its size. Mayer (1960) came to the conclusion that the time to plan and finish a project was 21 months. Those studies, even though supporting the fact that time is needed for an investment to start paying off, focus on how policies could be employed to strengthen production in an economy. In fact, the time for finishing a project is not specific and depends on the economy and the level of the currently available technology.

In the case of Bulgaria, the lag chosen is based on reasons connected with the bureaucracy of Bulgaria. Pre-building preparations and building permits could take up to 6 or 7 months, according to several private companies in the construction industry. Legislation on Public Procurement / Public Procurement Act could prolong the process by 3 to 6 months if purchases are worth more than BGN 100 000. Furthermore, Bulgaria is still a developing country, so, even if the physical capital is upgraded and new technology introduced, human capital still needs to be educated. Having Mayer’s calculation and these conditions in mind, we have decided to employ a two year lag of the FDI effect on TFP.

5. Empirical results

Our first step is to check the stationarity of the series as many macroeconomic series may contain a unit root due to using the Augmented Dickey–Fuller unit-root test. The test is based on the null hypothesis that a unit root exists in the series and, in order to continue with regressing the variables, we need to make the series stationary. We conduct ADF, assuming the existence of trend and drift and lag of 4, because of the serial correlation present. Results can be seen in Table 2.

All variables show unit roots; however, we take into consideration the differences in variables and simply readjust the test. For TFP, results show the existence of a trend, but no drift. Differencing does not solve the problem, so we employ the Hodrick–Prescott filter in order to get rid of the trend. Given the number of right-hand-side variables in the regression, and a sample of 40 observations, we cannot get sensible estimates when more than one lag is included. In addition, the HP filter approach is also chosen to make results comparable to other papers in the literature. Readjusting the ADF test and running it again results in stationarity of the variable.
For FDI, we have run ADF without trend, as it appears insignificant in the initial test, having received no unit root. The Research and Development, Government expenditure for Education and Government Protection expenditure variables show unit roots brought to stationarity using differencing. For Government spending on Health, differencing does not solve the problem, and, therefore, we employ the HP filter once again, which makes the series stationary.

**Table 1. Regression of TFP on FDI with 8th lag of FDI**

| Variable   | Coefficient | Std. Err | t      | P>|t| | 95% Conf. Interval |
|------------|-------------|----------|--------|-----|-------------------|
| L8.InFDI   | .147        | .0179    | 0.000  | .111 | .184   | .147              |
| _cons      | -1.307      | .192     | 0.000  | -1.7 | -.916  | -1.308            |

Number of observations =32  R-squared=0.692  Adj. R-squared =0.6817

**Table 2. Augmented Dickey-Fuller Test with 4 lags, trend and a constant**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mackinnon p-value</th>
<th>Trend p-value</th>
<th>Const p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>lnTFP</td>
<td>0.2856</td>
<td>0.011</td>
<td>0.155</td>
</tr>
<tr>
<td>lnFDI</td>
<td>0.1392</td>
<td>0.455</td>
<td>0.003</td>
</tr>
<tr>
<td>lnR&amp;D</td>
<td>0.2439</td>
<td>0.012</td>
<td>0.011</td>
</tr>
<tr>
<td>lnHealth</td>
<td>0.1501</td>
<td>0.034</td>
<td>0.007</td>
</tr>
<tr>
<td>lnEducation</td>
<td>0.6433</td>
<td>0.619</td>
<td>0.065</td>
</tr>
<tr>
<td>lnSocialWelfare</td>
<td>0.7569</td>
<td>0.179</td>
<td>0.098</td>
</tr>
</tbody>
</table>
We can start by running a regression of the pure model, as stated in equation (2). In order to receive meaningful results we take into consideration the FDI lag. Results of the regression of TFP on FDI (8th lag of FDI) can be seen in Table 1. Although we receive very promising results, supporting the claims discussed in the previous section, the regression is not full as it disregards most of the additional variables that could influence TFP.

In order to correct this problem we need to run the regression of equation (3), which takes this form:

\[
\ln TPF_t = a_0 + b_1 \ln FDI_{t-8} + b_2 \ln RD_t + b_3 \ln Health_t + b_4 \ln Education_t + b_5 \ln SocProt_t + \epsilon_t, \tag{5}
\]

with \(\ln TFP\) and \(\ln Health\) detrended through the HP filter and \(\ln RD, \ln Education\) and \(\ln SocProt\) differenced. In our regressions we incorporate the 8th lag of FDI, as already explained in the previous section. Results of the regression are presented in Table 3.

| Variable     | Coefficient | Std. Err | t  | P>|t|   | 95% Conf. Interval |
|--------------|-------------|----------|----|--------|-------------------|
| L8.\ln FDI   | 0.03        | 0.02     | 1.97 | 0.059 | -0.0014         | 0.069             |
| D1.\lnr_d    | -0.09       | 0.25     | -0.36 | 0.718 | -0.606          | 0.423             |
| Detr_\lnhealth | 0.325     | 0.135    | 2.41 | 0.023 | 0.05            | 0.623             |
| D1.\lnedu    | -0.22       | 0.19     | -1.15 | 0.262 | -0.627          | 0.172             |
| D1.\lnprot   | 0.607       | 0.244    | 2.49 | 0.019 | 0.106           | 1.108             |
| _cons        | -0.371      | 0.185    | -2.01 | 0.055 | -0.751          | 0.007             |

Number of observations = 32  R-squared = 0.4846  Adj. R-squared = 0.3855
It seems that government spending on social welfare, which is provided to households and individuals in need, actually has strong positive effect on TFP, which is counter-intuitive. An explanation for this might be that because the government provides funds to those unable to produce, their families have the freedom to focus on their work and be more productive. The research and development and educational expenditures appear insignificant and, if excluded from the regression, an Adjusted R-squared of 0.4004, significant FDI and coefficient of FDI of 0.0359764 are received. This coefficient is fairly low, but shows moderate correlation. Because of the low Adjusted R-squared, the model signals the existence of internal problems, most probably derived from the insufficient data. Nevertheless, we will accept the results since, even though they are close to 0, they are positive and establish a low threshold for the application of the model in the next section. The upper threshold of the model would be the pure regression of TFP on FDI, which we established in the beginning of the section, the result being 0.15\(^2\). Both results can be used to establish the effects of the model on the economy, and provide simulation evidence; if the implications of such evidence are ignored, this could lead to a distorted view of the growth path of the economy.

6. Simulations for Bulgaria

So far we have considered a model which shows positive dependency between TFP and FDI. However, we need to take our focus back and see the bigger picture when it comes to TFP and output. We have mentioned the Solow model and we have extensively discussed the Cobb-Douglass production function, so we cannot ignore one of their main applications, namely, finding the steady state of an economy. In this class of exogenous growth without exogenous exponential TFP growth, growth is monotonically decreasing along the transition path and approaches zero as the economy converges to its steady-state. Because TFP plays an essential part in the production function, we need to reconsider the model in its context.

In this section, we use a standard Ramsey (optimal) growth model, augmented using the FDI model described before, to compare the rate of convergence to an identical setup without FDI. We incorporate the TFP/FDI model in order to see whether an economy taking into consideration FDI would reach its steady state faster or more slowly. Results will show that there will be differences in terms of the time needed to reach the steady state in the presence of FDI augmentation and due to ignoring the FDI channel. In our simulation we employ optimisation techniques with respect to consumption and capital accumulation.

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2. We consider the estimate as an upper bound for the effect; therefore, results are to be taken with “a grain of salt.”
First, let us explain the optimisation methodology. The representative agent maximises total discounted utility, which is a function of consumption. In other words, the agent needs to select the optimal path of consumption over time or, alternatively, allocate output between consumption and capital accumulation (investment) over time. An equation stating these facts is as follows:

$$\max_{\{c_t\}} \sum_{t=0}^{\infty} b^t U(c_t), \quad (6)$$

where, $U(c_t)$ is the instantaneous utility function and $b$ is a discount factor such that $0 < b < 1$, as human beings consider consumption more valuable at early times than consumption further in the future.

The constraint to equation (6) is equation (7) depicting the aggregate consumption in the economy, which depends on the undepreciated capital stock remaining after the current period, the output produced in period $t$ from capital per worker using the technology for the period and the future capital stock:

$$c_t = (1 - \delta)k_t + Ak_t^{\alpha} - k_{t+1}, \quad (7)$$

The results of equation (6) have already been expressed in the Euler equation, which is a fundamental basis in intertemporal optimisation problems with dynamic constraints:

$$\frac{U'(c_t)}{bU'(c_{t+1})} = \frac{\alpha Ak_t^{\alpha - 1} + 1 - \delta}{1} \quad (8)$$

We can interpret equation (8) as the connection between intertemporal rate of substitution of consumption and marginal rate of transformation of capital. At steady state, consumption levels in period $t$ and period $t+1$ must be equal and, thus, utility throughout the periods must be constant:

$$1/b = \alpha Ak_{t+1}^{\alpha - 1} + 1 - \delta, \quad (9)$$

where $\alpha Ak_{t+1}^{\alpha - 1} - \delta$ represents the real return on investment after depreciation. Overall, the idea behind both equations (8) and (9) is that, in order for people to choose to invest, they need to receive an additional return or compensation in the next period in order for the utility to remain stable.

In connection to equation (8), we need to take into consideration the empirical fact for balanced growth, namely, that in order to have every component growing at the same rate, the utility function of consumption should be restricted to the CES (constant elasticity of substitution) form or:
and

\[ U'(c_t) = c_t^{-\sigma}. \]  

From here we can restate equation (8) to be:

\[ \frac{c_{t+1}}{c_t} = \left[ b (\alpha A k^{\frac{\alpha-1}{\alpha}}_{t+1} + 1 - \delta) \right]^{1/\sigma}, \]  

and we can easily express consumption in one period through consumption in the adjacent one. The intertemporal elasticity of substitution or \( 1/\sigma \) depends on how responsive the growth rate of consumption is to changes in real interest rate.

Finally, the so-called Transversality condition (TVC) has to be imposed. It is a boundary condition that rules out explosive paths, and guarantees stability of equilibrium paths for capital, consumption, investment and output. It necessitates that, at the end of the optimisation horizon, the discounted value of capital is zero.

\[ \lim_{t \to \infty} b^t U'(c_t) k_{t+1} = 0 \]  

In the long run we are expected to encounter the steady state where there is constant capital stock. This means that from equation (9) we can omit period identifications and rearrange the equation in order to receive the steady state capital value:

\[ k^* = \left[ \frac{1^\delta - 1}{\alpha A} \right]^{1/\alpha-1}. \]  

We assume that \( k_0 \) is given as a percentage of the steady state – we will assume that the economy starts at 10% of the steady state. We know what our optimal results are, so we are interested in the point of time at which we are going to reach these optimal results. We translate all of our findings and equations into a simulation that is going to show us at what point the Bulgarian economy is going to reach its steady state using the optimisation method for consumption. The simulation using MATLAB2015 and can be made available upon request.

In order to incorporate the model discussed in the previous sections we are going to assume that the capital of FDI is included in the overall capital and the problem is that economists are understating its influence on productivity and, therefore, do not account for it. This means that we have to restate our production function as:
\[ y_t = B k_t^{\alpha + \gamma}, \]  
\[ A = B k_t^\gamma \]  

The stationary parameters used are stated in Table 4, where \( b \) has been calculated on the basis of capital return equal to \( A A k_t^{\alpha - 1} = 1\% \) and the value for \( \sigma \) has been based on estimates of Hansen and Singleton (1983) and chosen to reflect the general tendencies of Bulgaria and the risk aversion of the population. TFP and are taken as averages from our previous findings and \( \gamma_1 \) and \( \gamma_2 \) are results from our econometric analysis. Nevertheless, the model can be easily adapted to simulate different economic variables. We also need to address the fact that \( B \) is calculated on the basis of \( k^* \), although it is present in the model including FDI. We have decided to do so because, in both simulations, the economy is converging towards the same steady state, but at different rates, which is what we are interested in.

**Table 4.** Parameters of simulation

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<tr>
<td>( A )</td>
<td>( \delta )</td>
<td>( \alpha )</td>
<td>( b )</td>
<td>( \gamma_1 )</td>
<td>( \gamma_2 )</td>
<td>( \sigma )</td>
</tr>
<tr>
<td>1.5</td>
<td>0.05</td>
<td>0.55</td>
<td>0.95</td>
<td>0.03</td>
<td>0.15</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Figure 2 and Figure 3 are a graphical representation of results. Both graphs show that the time needed to reach the steady state, considering the effects of FDI, is longer, which means that, by ignoring it, the standard Ramsey model is underestimating the actual time needed for the economy to converge to the steady state, no matter what the value of \( \gamma \) might be. The reason for this increase in time lies in the increased marginal product of capital when we consider the effects of FDI. This means that reinvestments and updates are necessary after a longer period, or smaller quantities, so as to increase the time of convergence. This result is also consistent with the delay observed before an investment becomes productive.
Figure 2. Application of the model: simulation with $\gamma_1$

Figure 3. Application of the model: simulation with $\gamma_2$
7. Summary and Conclusion

FDI is considered one of the levers that push an economy forward by increasing the productivity of a country. Relevant literature, however, still shows results, which, in some cases, indicate FDI actually has negative influence on TFP. This study argues that differences come from underspecified models, as well as econometric estimation problems, and aims at providing a stepping stone for further development of policies and programmes to attract FDI. In order to solve such problems, we analyse Bulgarian data for the period 2004-2013, employing a model that assumes that TFP or increase in knowledge is a function of new capital or FDI (learning-by-doing approach).

In our study, we have provided a model that not only studies the relationship between FDI and TFP, but also incorporates additional variables in empirical results, which might influence the aforementioned relationship. By doing so, we find evidence that FDI has a positive influence on TFP in the way that the model proposes. We find that, in fact, FDI has a lagged effect on TFP, which could easily change the way policy makers see foreign investments and their effects.

We come to the conclusion that FDI influences TFP in a positive way, but not in a strong manner. We can speculate on what the reason for this could be and the most apparent one is that Bulgaria is still a developing country and does not yet have the proper channels so as to take full advantage of incoming investments. The fact that not every industry in the country receives investments from abroad might limit the actual inflow, which could further influence empirical results and decrease the influence on TFP. Of course, we cannot ignore the fact that data available for research are limited so empirical results might incorporate consequences due to lack of information.

We conclude our study by applying the augmenting qualities of FDI to an optimal growth model, in order to find the effects on the growth path of the economy along the way to convergence to its steady state. Results unequivocally show that no matter what the value of the effect of FDI on TFP may be, the rate of convergence, in comparison to that when FDI is not accounted for, appears longer. Thus we reach the conclusion that, by ignoring FDI effects, the standard optimal growth model distorts the view of the economy and presents an unrealistic time frame.

By using these findings, the reader should be able to better understand the important role of Foreign Direct Investment for productivity in Bulgaria. By revealing the relationship between FDI and TFP, policy makers, politicians, as well as government officials and economists should be able to re-evaluate their positions regarding capital from abroad. We hope that findings similar to ours would encourage future studies on the topic, as well as positive development of the Bulgarian international standing regarding FDI. We firmly believe that facilitating the ease of assimilation of foreign capital would boost the economy and would positively influence future improvement of the country.
Appendix 1: Capital

The Gross Domestic Product published by the National Statistical Institute (NSI) has been used as a measure of $Y$, and the hours worked by the persons employed, which are also published by (NSI), as a measure of $L$. Data on $K$ are not published and, therefore, it is additionally calculated through the ‘perpetual inventory method’ or:

$$K_t = I_t + (1 - \delta)K_{t-1}$$

In this equation, $I_t$ represents total current investment and $\delta$ the depreciation rate. A problem occurs in the calculation of the initial capital - $K_0$. The method used for calculating initial capital is described in equation (2) (Hall et al., 1999) - initial capital equals the ratio of initial investment to depreciation rate. For initial capital, we use the gross fixed capital formation and 5% depreciation rate (Ganev, 2015).

$$K_0 = \frac{I_0}{\delta + g}$$

We assume that growth rate of investments $g$ in long periods is 0, because of high volatility in the years between 1991 and 2014 (Ganev, 2015).

In order to be able to calculate TFP we also need to find the values of $\alpha$ and $\beta$. We take advantage of the assumption that $\alpha + \beta = 1$, thus we need to find one only of the two. We use the ratio of Labour cost to GDP in order to receive $\alpha$. Labour cost is calculated as the average wage per hour multiplied by the hours worked in the year. The average wage is in 2010 prices and is taken from the National Statistical Institute.

Appendix 2: Seasonality adjustment

Seasonality adjustment of data is performed by using a centred moving average. The only data that this method used is Labour. We deal with quarterly data, so the periodic effect has a period of 4 observations. We calculate centred moving averages for each observation (excluding the first and last 2) following the formula:

$$CMA = \frac{Y_{n-2}}{8} + \frac{Y_{n-1} + Y_n + Y_{n+1} + Y_{n+2}}{4}$$

which represents our centred moving average for the first 5 observations.

We continue by calculating ratios between each observation and its CMA. This shows us how the observation varied from the CMA. We then calculate the 4 quarterly unadjusted seasonal indices – each represents an average of the ratios for each quarter in each year (the average of all first quarter ratios, the average of all second quarter ratios, etc.) and these will give us average deviation percentages for each quarter in our data. We divide each of the indices by the average of the four adjusted seasonal indices received so as to prevent any statistical errors. In order to finally receive our de-seasonalised data, we divide each observation by its respective adjusted seasonal index.
References


