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ON THE INTERPLAY BETWEEN INTERGENERATIONAL TRANSFERS AND NATURAL RESOURCES

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Abstract
This paper studies an overlapping generations model with selfish agents, natural resources and human capital externalities. The initial result is to quantify the economic effects of intergenerational transfers by comparing a complete markets allocation with transfers to an allocation without transfers due to incomplete markets. The core contribution is then to show that a higher resource regeneration rate boosts the effect of transfers on economic growth for both allocations, although it also implies a higher gap in growth performances between them. Finally, it is shown that transfers can be financed through a constant lump-sum tax relative to the output level.

JEL Classification: H23, H55, Q32
Key words: Overlapping generations, intergenerational transfers, natural resources, endogenous growth

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1. Introduction

In the last few decades, large investments have been carried out around the globe in order gradually to satisfy higher fractions of energy demand through more renewable natural resources. At the same time, economies abundantly endowed with exhaustible resources are preparing themselves to face the challenge of upcoming resource depletion. As an example of this, large policy debates started in resource-rich countries (e.g. Norway, Venezuela) as regards the long-term sustainability of social security policies (Harding & van der Ploeg, 2013). In light of these facts, the impact of the degree of resource renewability on modern economies −and more precisely on intergenerational transfers− qualifies as a relevant economic question.

The paper analyzes this topic in a model of endogenous growth for a closed economy with overlapping generations of selfish agents, natural resources and human capital externalities. The initial result is to quantify the effects of intergenerational transfers on economic growth. The core contribution of the paper is then to investigate whether and how the degree of resource renewability influences the economic effects of intergenerational transfers. Intergenerational transfers consist of transfers that a given cohort bestows upon a different cohort (either voluntarily or as a result of a transfers scheme implemented by a planner): unlike bequests motivated by dynastic altruism, such transfers reflect the existence of potential individual gains from trading with adjacent generations (e.g. young and old agents may increase private utility by trading education versus health care, long-lasting assets versus pensions) and may arise as political equilibria (Sjoblom, 1985) or as Pareto-improving intergenerational contracts (Esteban and Sakovics, 1993) in the standard overlapping generations model with finite lives and selfish agents (Boldrin and Rustichini, 2000; Rangel, 2003; Boldrin and Montes, 2005).

The following is a brief summary of the analysis. At first, the benchmark framework of the competitive economy with complete markets (Complete Markets Allocation, CMA hereafter) and intergenerational transfers is developed. Consider a simple three-period economy in which young agents overlap with adult workers and retired old agents. A stock of resources is necessary[1] for producing consumption goods and is initially owned by old agents under full property rights: resource assets are transmitted to the adult generation via a standard competitive market.[2] As a

1. A resource is defined as *necessary* if output is zero whenever the quantity of the resource used in production is zero, i.e. $F(H, 0) = 0$.

2. In this environment, sustained consumption is not guaranteed: the market valuation of oil assets

factor of production, natural resources differ in essence from physical capital inasmuch as they are destroyed and get depleted rather than being accumulated through investment. Although natural resources that are essential for production are scarce, growth can be sustained through the accumulation of human capital that spills over across generations in the form of public knowledge. However, the knowledge stock only grows if old agents invest in the education of the newborn generation. This mechanism creates potential gains for intergenerational transfers independently of the problem of resource scarcity. An intergenerational transfer scheme financing education and pensions is therefore implemented by agents in the decentralized economy with complete markets (CMA). This benchmark framework is subsequently simulated and compared with the case of incomplete markets allocation (IMA, hereafter) without transfers. This comparison highlights the substantial positive effects of intergenerational transfers on economic growth and constitutes the first contribution of this paper.

The main contribution is summarized as follows. Intuitively and other things being equal, a higher resource regeneration rate would imply that one (natural capital) of the two productive factors of the economy (the other being labor) becomes more widely available. In turn, this would determine a factor reallocation away from labor and would call for a lower degree of human capital accumulation. If this were the mechanism at work, the higher regeneration rate would translate into a lower fraction of intergenerational transfers with respect to output (since education transfers are necessary to accumulate human capital). Generally, the intention is to fully investigate which mechanisms tend to prevail as a response to a change in the degree of resource renewability. The results show that a higher resource regeneration rate magnifies the volume and the positive effects of transfers on growth performances. This happens through the regeneration rate's effects on the rate of return on resource wealth, resource use and output growth rate. Last but not least, from the analysis of the revenue side it is shown that the planner can replicate the CMA growth performances by implementing a period-by-period intergenerational

limits the rate of depletion of the oil stock only to the extent that selling oil assets to the adults is actually profitable to the retiring agent. In other words, future generations may be bound to experience declining consumption levels because the distribution of resource wealth is inevitably biased in favor of the "first father". More generally, neither sustainability nor resource preservation are guaranteed when natural capital is private property. This result holds in general equilibrium models with infinitely lived agents (Dasgupta and Heal, 1974), and is furthermore valid when assuming selfish agents with finite lifetimes (Mourmouras, 1991, 1993).

transfers scheme financed through lump-sum taxation.[3] It is finally also shown that such a transfer policy can be financed through a constant lump-sum tax relative to the output level.

The paper proceeds as follows: Section 2 introduces the model. The potential gains from implementing intergenerational transfers are shown by comparing the efficiency properties of the CMA (developed in Section 3) with the IMA (developed in Section 4) in which neither education to the young nor pay-as-you-go pensions to the old are implemented. In Section 5, I address the issue of how resource renewability affects the features of intergenerational transfers by simulating the model under different parameterizations and by implementing different taxation instruments. Section 6 concludes.

2. The Model

2.1 Production, Resources and Human Capital

The production sector consists of an indefinite number of competitive firms that produce a homogeneous consumption good using human capital and a primary resource under constant returns to scale. Aggregate output is denoted by Y_t in physical terms and is the numeraire good. The technology is a well-behaved production function displaying strictly decreasing marginal productivities and satisfying the Inada conditions:

$$Y_t = F\left(H_t, X_t\right), \tag{1}$$

where $t = \{0,1,2,\ldots\infty\}$ is the period index in discrete time, $H_t \equiv h_t \ell_t$ is aggregate human capital – given by the current stock of knowledge per worker h_t times the number of workers ℓ_t –and X_t is resource use– i.e., a flow of primary resource extracted from a natural stock and destroyed in the production process. Denoting the wage rate by w_t and the resource price by p_t, the profit-maximizing conditions imply:

$$w_t = F_{H_t} \quad \text{and} \quad p_t = F_{X_t}, \tag{2}$$

where constant returns to scale ensure zero profits in the final sector.

With respect to resource use, the analysis is inspired by the sustainability liter-

3. This result draws in part on Boldrin and Montes (2005), in which public financing of education and pensions is shown to be implemented by a transfer scheme and financed through a lump-sum tax. However, the benchmark framework in Boldrin and Montes (2005) differs from the current model since they do not contemplate the use of natural resources as factor of production.

ature (Dasgupta and Heal, 1974; Solow, 1974; Stiglitz, 1974): economic growth may be negative as a result of natural scarcity – a scenario that is surely compelling when the resource stock is non-renewable. However, for the sake of generality, I allow for the possibility of natural regeneration: the resource stock R_t obeys the dynamic law

$$R_{t+1} = \left(R_t - X_t\right)\left(1+\gamma\right), \tag{3}$$

where $\gamma \geqslant 0$ is a constant marginal rate of biological renewal. Setting $\gamma = 0$, equation (3) reduces to the standard law for exhaustible resources like oil and minerals. In each period, the part of the resource stock that is not destroyed in production constitutes resource assets, A_t, that will be used for production in the future: agents exchange shares of A_t on a perfectly competitive financial market. The resource stock thus equals, in each period:

$$R_t = X_t + A_t. \tag{4}$$

The dynamics of aggregate human capital $H_t \equiv h_t \ell_t$ and of individual knowledge h_t are linked to the demographic structure, which comprises three overlapping generations: in each period $t = \{0,1,2,\ldots\infty\}$, there are N_t^y young, N_t^a adult and N_t^o old agents, with a constant exogenous population growth rate:

$$N_{t+1}^y / N_t^y = N_{t+1}^a / N_t^a = N_{t+1}^o / N_t^o = 1+n. \tag{5}$$

For simplicity, young agents do not work and do not consume: in the first period of life, t, each agent studies to acquire h_{t+1} units of knowledge that will determine her productivity as a worker during adulthood. In the second period of life, t+1, agents supply inelastically their human capital to the production sector. In the third period of life, t+2, agents do not work. The number of workers thus coincides with the mass of adults, $\ell_t = N_t^a$. Each worker's knowledge is determined by the generic learning technology

$$h_{t+1} \equiv \eta\left(h_t, \varepsilon_t\right), \qquad \frac{\partial \eta_t}{\partial \varepsilon_t} > 0, \tag{6}$$

where ε_t is the economy's propensity to spend on education. Denoting by e_t the investment per young person in education expressed in units of final output, and by $y_t = Y_t / N_t^y$ output per young person, the education spending share can be rewritten as

$$\varepsilon_t \equiv \frac{e_t}{y_t}. \tag{7}$$

Assume further that knowledge grows if and only if the economy spends a positive amount of output on education: when ε_t=0, human capital is constant at the previous level (for simplicity, we rule out knowledge depreciation over time). A convenient function that incorporates this hypothesis is

$$h_{t+1} = \eta\left(h_t, \frac{e_t}{y_t}\right) = h_t \cdot \left(1 + \mu \frac{e_t}{y_t}\right), \tag{8}$$

Where $\mu > 0$ is a constant exogenous parameter reflecting the marginal impact of the propensity to spend on education for future knowledge growth.

2.2 Household Behavior

Assume that agents are homogeneous, endowed with perfect foresight, and selfish: each agent seeks to maximize her own lifetime utility. To simplify the discussion, the consumer problem is split into two logical steps. First, agents decide how to trade resource assets with adjacent generations in order to maximize the present-value of net income from resource ownership. Second, the consumer decides how to allocate total lifetime income between consumption and savings when adult, and consumption when old. The reason for this distinction is that I will consider different variants of step two.

Considering the first step, the typical adult in period t saves in the form of resource assets – i.e., she purchases from old agents a fraction $1/N_t^a$ of A_t at unit price q_t. In period t+1, the same agent is old, she owns a fraction $1/N_{t+1}^o$ of the existing resource stock R_{t+1}, and sells X_{t+1}/N_{t+1}^o units as "resource use" to production firms at price p_{t+1}, and A_{t+1}/N_{t+1}^o units as "resource assets" to adult agents at price q_{t+1}. Consequently, the present value of net resource incomes over the life-cycle is

$$\frac{1}{N_t^a} \cdot \left(\frac{q_{t+1}A_{t+1} + p_{t+1}X_{t+1}}{1+i_{t+1}} - q_t A_t\right), \tag{9}$$

where i_{t+1} is the implicit rate of return on resource wealth. Given the resource constraints (3) and (4), the maximization of net resource income implies two basic conditions of no arbitrage (see the Mathematical Appendix for details). On the one hand, there must be price equalization between resource assets and resource use, p_t=q_t, in each period. On the other hand, the dynamics of resource rents must satisfy

the generalized Hotelling rule

$$\frac{p_{t+1}}{p_t} = \frac{q_{t+1}}{q_t} = \frac{1+i_{t+1}}{1+\gamma}, \tag{10}$$

whereby the resource price grows (declines) over time if the rate of return exceeds (falls short of) the regeneration rate. If the natural stock is non-renewable, $\gamma = 0$, expression (10) declines to the standard Hotelling's (1931) rule, according to which the resource price must grow at the rate of return in order to guarantee intertemporal no-arbitrage. These results allow us to define individual savings during adulthood as $s_t = q_t A_t / N_t^a$, associated to the gross return ($1+i_{t+1}$) in the subsequent period of life.

In the second step, agents decide how to allocate total lifetime income between consumption and savings in order to maximize lifetime utility. Because young agents do not work and do not consume, preferences are defined over two periods only: the lifetime utility of the young born in period $t-1$ reads

$$u_{t-1}(c_t, d_{t+1}) = v(c_t) + \beta \cdot v(d_{t+1}), \tag{11}$$

where c_t is consumption when adult, d_{t+1} is consumption when old, $\beta \in (0,1)$ is the private discount factor, and $v(\cdot)$ is a well-behaved utility function implying positive and strictly decreasing marginal utility, and satisfying the Inada conditions.

Importantly, I can consider different specifications of the consumer problem depending on the structure of credit markets and the possible existence of inter-generational transfers. The benchmark scenario (CMA) is a world in which there are complete credit markets: young agents are able to borrow in their first period of life the amount of output they wish to invest in education and they repay the debt at the prevailing interest rate during adulthood. The alternative scenario is represented by a world in which credit markets for education financing are missing, and the accumulation of human capital hinges on the existence of intergenerational transfers (IMA). In the next section, I present the complete markets allocation (CMA).

3. The case of complete markets

Suppose that there exist complete credit markets for education financing: each young agent born in period $t-1$ borrows b_{t-1} units of output for financing her education investment e_{t-1}, and repays the amount $b_{t-1}(1+i_t)$ during adulthood. Accordingly, the N_t^a adults in period t finance current education investment $N_t^y b_t$ and receive, in the aggregate, $N_t^y b_t (1+i_{t+1})$ units during old age. Consequently, the

typical consumer maximizes utility (11) subject to

$$c_t = w_t h_t - s_t - b_{t-1}\left(1+i_t\right) - b_t\left(1+n\right), \tag{12}$$

$$d_{t+1} = s_t\left(1+i_{t+1}\right) + b_t\left(1+n\right)\left(1+i_{t+1}\right), \tag{13}$$

$$h_t = \eta\left(h_{t-1}, e_{t-1}/y_{t-1}\right) \quad \text{with} \quad e_{t-1} = b_{t-1}, \tag{14}$$

where (12) and (13) are the budget constraints for the second and third period of life, respectively, and (14) is the knowledge accumulation constraint where education investment is determined by the initial debt, and the knowledge and labor productivity of the previous generation (h_{t-1} and y_{t-1}) are taken as given. The solution to this problem yields the conditions (see the Mathematical Appendix):

$$\frac{v'\left(c_t^\star\right)}{\beta v'\left(d_{t+1}^\star\right)} = 1 + i_{t+1}^\star, \tag{15}$$

$$w_t^\star \cdot \frac{\partial \eta\left(h_{t-1}, e_{t-1}^\star / y_{t-1}\right)}{\partial e_{t-1}^\star} = 1 + i_t^\star, \tag{16}$$

where superscript '$\star$' denotes equilibrium variables in the CMA, equation (15) is the usual Euler condition for consumption growth, and equation (16) establishes that the marginal private benefit from education investment –that is, the increase in wage income generated by higher individual knowledge– must match the private marginal cost of borrowing in the first period of life. Combining equations (15, 16) with the profit-maximizing conditions in the production sector (2) and the Hotelling rule (10), we can characterize the equilibrium arising under complete credit markets. It is essential to provide a full characterization of the CMA when preferences and technologies take simple forms that yield neat solutions for the equilibrium path. Consider the following:

Log-linear model *Production equals* $F\left(H_t, X_t\right) = H_t^\alpha X_t^{1-\alpha}$*, and static utility is* $v(\cdot)=\ln(\cdot)$.

In the log-linear model, the simplifying role of Cobb-Douglas production technology and logarithmic utilities is obvious and well-established in the growth literature: output, resource use and human capital will all grow at constant growth rates in each period $t = \{0,1,2,\ldots\infty\}$. The competitive equilibrium with complete markets is characterized as follows (full derivation of equilibrium characterization in the Mathematical Appendix):

Remark 1 *A competitive equilibrium with complete markets is defined by (10, 15, 16); by the aggregate constraint and focs (17); by the propensities to invest in human capital (18) and to consume (19); by the equilibrium interest rate (20) and by the growth rates of aggregate human capital (21), resource use (22) and output (23):*

$$F\left(H_t^\star, X_t^\star\right) = N_t^y e_t^\star + N_t^a c_t^\star + N_t^o d_t^\star, \qquad w_t = F_{H_t} \quad \text{and} \quad p_t = F_{X_t}, \tag{17}$$

$$\frac{N_t^y e_t^\star}{Y_t^\star} = \frac{1}{\mu} \cdot \frac{h_{t+1}^\star - h_t^\star}{h_t^\star}, \tag{18}$$

$$\frac{N_t^a c_t^\star}{Y_t^\star} = \frac{\alpha}{(1+\beta)(1+n)\left(1+\mu\varepsilon^\star\right)}, \quad \frac{N_t^o d_t^\star}{Y_t^\star} = \frac{\beta\alpha}{(1+\beta)(1+n)^2\left(1+\mu\varepsilon^\star\right)^2}, \tag{19}$$

$$\left(1+i^\star\right) = \left(\alpha\mu\right)^\alpha \left(1+\gamma\right)^{1-\alpha}, \tag{20}$$

$$\frac{H_{t+1}^\star}{H_t^\star} = (1+n)\left(1+\mu\varepsilon^\star\right), \tag{21}$$

$$\frac{X_{t+1}^\star}{X_t^\star} = \frac{1+\gamma}{\alpha\mu}(1+n)\left(1+\mu\varepsilon^\star\right), \tag{22}$$

$$\frac{Y_{t+1}^\star}{Y_t^\star} = \left(\frac{1+\gamma}{\alpha\mu}\right)^{1-\alpha}(1+n)\left(1+\mu\varepsilon^\star\right). \tag{23}$$

This equilibrium defines the dynamic system $\Phi : (e_{t-1}^\star, h_t^\star, y_t^\star, X_t^\star) \mapsto (e_t^\star, h_{t+1}^\star, y_{t+1}^\star, X_{t+1}^\star)$. *Given initial conditions* $(e_{t-1}^\star, h_{t-1}^\star, y_{t-1}^\star, X_0^\star)$ *and parameters* $(\alpha, \beta, \mu, n, \gamma)$, *the system* Φ *evolves along the equilibrium path* $\left\{(e_t^*, h_{t+1}^*, y_{t+1}^*, X_{t+1}^*)\right\}_{t=0}^{\infty}$; *given this equilibrium path all remaining factor prices and quantities are determined.*

As is shown in (20) in Remark 1, the equilibrium interest rate factor $\left(1+i^\star\right)$ turns out to be a constant weighted average of the regeneration rate γ and of the marginal impact of the propensity to spend on education μ. The intuition behind this result is that a higher regeneration rate γ would allow a more sustainable resource use, postponing stock depletion and thereby contributing to providing a higher gross return on resource wealth.

4. The incomplete markets allocation

In this section the allocation with incomplete markets (IMA) is developed. Suppose

therefore that credit markets for education financing are missing or incomplete: young agents are not able to borrow to finance their education. Consequently, intergenerational transfers do not take place and the economy moves to an inefficient equilibrium. More precisely, (16) is now violated and profitable investment in human capital cannot any longer be achieved. The typical consumer will now maximize utility (11) subject to

$$c_t = w_t h_t - s_t, \tag{24}$$

$$d_{t+1} = s_t\left(1+i_{t+1}\right), \tag{25}$$

$$h_t = h_{t-1}, \tag{26}$$

where (24) and (25) are the budget constraints for the second and third period of life, respectively, and (26) is the knowledge accumulation constraint anchored at a given past value. The solution to this problem yields the following conventional Euler condition for consumption growth (see the Mathematical Appendix):

$$v'\left(c_t\right) = \beta\left(1+i_{t+1}\right)v'\left(d_{t+1}\right). \tag{27}$$

Assuming a log-linear model as in the CMA case and combining equation (27) with the profit-maximizing conditions in the production sector (2) and the Hotelling rule (10), we can characterize the equilibrium arising under incomplete credit markets. Let us start to observe that, absent any educational expenditure allowing the young generation to invest in education and accumulate human capital, the growth rate of human capital will no longer be endogenously determined by the model:

$$\frac{H_{t+1}^{\#}}{H_t^{\#}} = \frac{h_{t+1}\ell_{t+1}}{h_t\ell_t} = \left(1+n\right)\frac{h_{t+1}}{h_t} = (1+n), \tag{28}$$

where superscript '#' denotes equilibrium variables in the IMA. The absence of intergenerational transfers due to incomplete markets determined a slow-down in human capital accumulation with respect to the CMA allocation, creating a source of inefficiency. Human capital accumulation is however still positive due to population growth. The equilibrium characterization for the IMA is summarized as follows (full derivation of equilibrium characterization in the Mathematical Appendix):

Remark 2 *A competitive equilibrium with incomplete markets is defined by (10, 27); by the aggregate constraint and focs (29); by the propensities to consume (30); by the equilibrium interest rate (31) and by the growth rates of aggregate human*

capital (32), resource use (33) and output (34):

$$F_t(H_t^{\#}, X_t^{\#}) = N_t^a c_t^{\#} + N_t^o d_t^{\#}, \qquad w_t = F_{H_t} \quad \text{and} \quad p_t = F_{X_t}, \tag{29}$$

$$\frac{N_t^a c_t^{\#}}{Y_t^{\#}} = \frac{\alpha}{1+\beta}, \qquad \frac{N_t^o d_t^{\#}}{Y_t^{\#}} = \frac{1+\beta-\alpha}{1+\beta}, \tag{30}$$

$$\left(1+i^{\#}\right) = \left[\frac{(1+\beta-\alpha)(1+n)}{\alpha\beta}\right]^{\alpha} (1+\gamma)^{1-\alpha}, \tag{31}$$

$$\frac{H_{t+1}^{\#}}{H_t^{\#}} = (1+n), \tag{32}$$

$$\frac{X_{t+1}^{\#}}{X_t^{\#}} = \frac{\alpha\beta}{1+\beta-\alpha}(1+\gamma), \tag{33}$$

$$\frac{Y_{t+1}^{\#}}{Y_t^{\#}} = (1+n)^{\alpha}\left[\frac{\alpha\beta(1+\gamma)}{1+\beta-\alpha}\right]^{1-\alpha}. \tag{34}$$

This equilibrium defines the dynamic system $\Omega : (h_t^{\#}, y_t^{\#}, X_t^{\#}) \mapsto (h_{t+1}^{\#}, y_{t+1}^{\#}, X_{t+1}^{\#})$. *Given initial conditions* $(h_{t-1}^{\#}, y_{t-1}^{\#}, X_0^{\#})$ *and parameters* $(\alpha, \beta, n, \gamma)$, *the system* Ω *evolves along the equilibrium path* $\left\{(h_{t+1}^{**}, y_{t+1}^{**}, X_{t+1}^{**})\right\}_{t=0}^{\infty}$; *given this equilibrium path all remaining factor prices and quantities are determined.*

As is shown in (31), this time the interest rate factor ($1+i^{\#}$) turns out to be a constant weighted average of the regeneration rate γ and of the population growth rate n. Relatively to the CMA benchmark allocation, the impossibility of financing education limits human capital accumulation and has detrimental effects on the long-run growth scenario for the economy. An estimation of the magnitude of this detrimental effect will be computed in the next section on resources and intergenerational transfers.

5. Resources and intergenerational transfers

The scope of this section is threefold. At first, the dynamics of resource use is presented (Section 5.1). Later on, I proceed by simulating the gap in growth performances between the CMA and the IMA induced by intergenerational transfers and the effects on this gap induced by a higher resource regeneration rate (Section 5.2). In conclusion, the interaction of lump-sum taxation with resource regeneration rate and growth performances is explored (Section 5.3).

5.1 Resource use dynamics

Before specifying parameters and initial conditions of the calibration exercise, let us show the framework in which the different levels of the regeneration rate γ will be inserted. Starting from the accumulation law (3) and iterating the stock equation we obtain:

$$R_T = R_0(1+\gamma)^T - \sum_{t=0}^{T-1} X_t(1+\gamma)^{T-t}. \tag{35}$$

Next, consider the resource use dynamics: for CMA and IMA we have that X_t grows at constant rates which we redefine as $\Delta^\star$ and $\Delta^\#$. Therefore, we have in general that $X_{t+1} = \Delta \cdot X_t$ so that $X_t = \Delta^t \cdot X_0$. Substituting this into the resource constraint and rearranging we have

$$R_T(1+\gamma)^{-T} = R_0 - X_0 \sum_{t=0}^{T-1} \left(\frac{\Delta}{1+\gamma} \right)^t. \tag{36}$$

An important condition to be imposed is the following transversality condition, not allowing the stock to grow faster than the regeneration rate:

$$\lim_{T\to\infty} R_T(1+\gamma)^{-T} = 0, \tag{37}$$

from which we get

$$R_0 = X_0 \lim_{T\to\infty} \sum_{t=0}^{T-1} \left(\frac{\Delta}{1+\gamma} \right)^t = X_0 \frac{1}{1-\frac{\Delta}{1+\gamma}}. \tag{38}$$

Note that in order to obtain a solution we must have $\frac{\Delta}{1+\gamma} < 1$. This is certainly true in the IMA since from (33) we observe that $\frac{\Delta^\#}{1+\gamma} = \frac{\alpha\beta}{1+\beta-\alpha} < 1$. In the CMA equilibrium instead, (22) implies that $\frac{\Delta^\star}{1+\gamma} = \frac{(1+n)(1+\mu\varepsilon^\star)}{\alpha\mu}$, which satisfies $\frac{\Delta^\star}{1+\gamma} < 1$ if and only if

$$\alpha\mu > (1+n)(1+\mu\varepsilon^\star). \tag{39}$$

This inequality has to be strictly verified in order for the model to exhibit a solution. In other words, if and only if the joint choice of parameters ($\varepsilon^\star$, μ) in the parameterization below satisfies the inequality (39), we can finally obtain the initial rate of resource use for all allocations:

$$X_0 = \frac{1+\gamma-\Delta}{1+\gamma} R_0. \quad \textbf{(40)}$$

Given endowments R_0,H_0, and the CMA and IMA's resource use dynamics $\Delta^{\star}$ and $\Delta^{\#}$, we can therefore obtain the initial values of X_0 and output since $Y_0 = F(H_0, X_0)$. After that, it will be possible to compare the gaps between the constant growth rates of output for the different allocations.

5.2 A simulation of the economic effects of transfers

This subsection calibrates the model in order to simulate the economic effects of transfers when a jump in the resource regeneration rate occurs. The model is parameterized as if each time interval $(t, t+1)$ would be equal to 25 years. This time span is realistically sufficient to allow the young generation at *t* to grow adult at *t*+1 and later on to become old at *t*+2. As regards the share of human capital in production, assume α=0.85. The corresponding share of resources is then (1−α)=0.15.[4] In order to obtain an implicit corresponding annual level of β=0.98 we have to set the generational private discount factor at β=0.61. The growth of population is set without loss of generality at the lower bound *n*=0. For both allocations, the initial level of human capital has been arbitrarily set at H_0=10, whilst the initial resource stock is endowed with an amount of resource given by R_0=100.[5]

As regards the resource regeneration rate $\gamma \geq 0$, I intend to compare two cases:

$$\gamma_{low} = 0.5, \quad \textbf{(41)}$$

$$\gamma_{high} = 0.8. \quad \textbf{(42)}$$

In other words, I intend to evaluate the effects of a 60 percent increase in the

4. α=0.85 and (1−α)=0.15 are commonly used shares within the literature of growth models for resource-rich economies with no physical capital (Valente, 2008, 2011).

5. A numerical exercise provided the range of values for $(\varepsilon^{\star}, \mu)$ which satisfies simultaneously the stationarity condition for $\varepsilon^{\star}$ given in the Mathematical Appendix and the inequality given in (39). The Maple code for this numerical exercise is available on request. Within this range of values for $(\varepsilon^{\star}, \mu)$, the pair ($\varepsilon^{\star} = 0.012577$; $\mu = 1.8$) was chosen in order to prioritize acceptable values for the equilibrium interest rates.

resource regeneration rate on intergenerational transfers and growth performances.[6] The following tables present and summarize the results,[7] which are discussed in more detail below:

CMA	$i^{\star}$	Annual $i^{\star}$	$X_0^{\star}$	$g^{\star}$
$\gamma_{high} = 0.8$	0.5255	0.0170	55.44	0.0196
$\gamma_{low} = 0.5$	0.5678	0.0181	33.16	0.0478

(43)

IMA	$i^{\#}$	Annual $i^{\#}$	$X_0^{\#}$	$g^{\#}$
$\gamma_{high} = 0.5$	0.4709	0.0156	54.51	0.0035
$\gamma_{high} = 0.5$	0.5116	0.0167	31.77	0.0312

(44)

in which $g^{\star} = \frac{Y_{t+1}^{\star}}{Y_t^{\star}} - 1$ and $g^{\#} = \frac{Y_{t+1}^{\#}}{Y_t^{\#}} - 1$.

5.2.1 The case of $\gamma_{low} = 0.5$

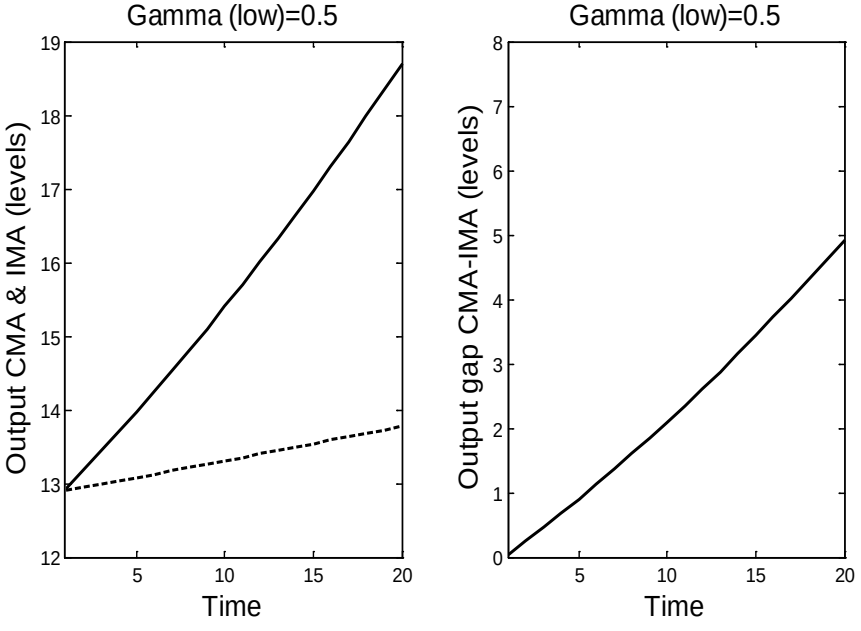
At first, let us verify that the allocations indeed exhibit interior solutions. The constant values for the interest rates (time indexes are thereby dropped) for the CMA and IMA allocations are given respectively by $i^{\star} = 0.5255$ and $i^{\#} = 0.4709$, ensuring the presence of interior solutions. These are generational interest rates on a time span of 25 years and they correspond respectively to $i^{\star} = 0.0170$ and $i^{\#} = 0.0156$ on a yearly basis. Following the procedure presented in the previous subsection 5.1, endowments R_0 and H_0 together with the resource-use dynamics in the CMA and IMA allow us to obtain the initial values of X_0, given by $X_0^{\star} = 55.44$ and $X_0^{\#} = 54.51$. Subsequently, by recalling that $Y_0 = F(H_0, X_0)$, we can simply simulate output time series. The following Figure 1 shows output time series (in levels,

6. A few more words are needed in order to fully justify the choice of these specific regeneration rate levels. Note that the above mentioned values for the parameters α, β, n imply that the IMA exhibits approximately negative output growth for $\gamma \leq 0.46$. The lower threshold of the resource regeneration rate has therefore been set at γ_{low}=0.5 in order to make a comparison between allocations implying exclusively positive growth rates.

7. The MATLAB code used for this simulation is available from the author on request.

left-hand side) for CMA (solid line) and IMA (dashed line) and the positive output gap (also in levels, right-hand side) that intergenerational transfers create between the two allocations:

Figure 1: The case of γ_{low}



More interestingly, we can observe the results in terms of constant growth rates. The constant growth rate of income for the CMA allocation is given by $g^{\star} = 0.0196$, whilst for the IMA we have $g^{\#} = 0.0035$. As expected, the poorer growth performance of the IMA is due to the impossibility of financing education and the subsequent limited growth in human capital accumulation. This initial result can be summarized as follows:

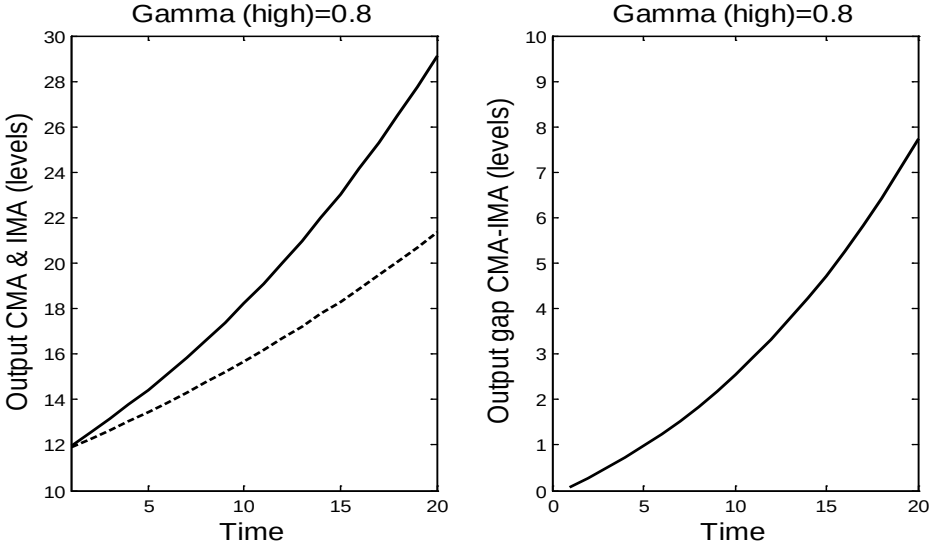
Remark 3 *Given the regeneration rate level γ_{low}, the economic effects of intergenerational transfers are estimated by the positive gap in growth rates given by the difference between $g^{\star}$ and $g^{\#}$.*

Let us now move to the alternative scenario and observe how these growth rates and the transfers scheme will respond to a higher regeneration rate.

5.2.2 The case of $\gamma_{high} = 0.8$

Higher resource regeneration rate γ implies at first that the rate of return on resource wealth is increased. The constant values for the interest rates are now given respectively by $i^{\star} = 0.5678$ and $i^{\#} = 0.5116$, that correspond respectively to $i^{\star} = 0.0181$ and $i^{\#} = 0.0167$ on a yearly basis. In addition, the initial values of X_0 are now given by $X_0^{\star} = 33.16$ and $X_0^{\#} = 31.77$ indicating a more sustainable resource depletion path than under γ_{low}. As anticipated above, the intuition behind these results is that a higher regeneration rate γ makes it possible to postpone resource depletion and thereby contributes to providing a higher gross return on resource wealth, as can be observed in both (20) and (31). The following Figure 2 shows the output series (left-hand side, again solid line for the CMA and dashed for the IMA) and the output gap (right-hand side) between the two allocations, for the case of γ_{high}:

Figure 2: The case of γ_{high}

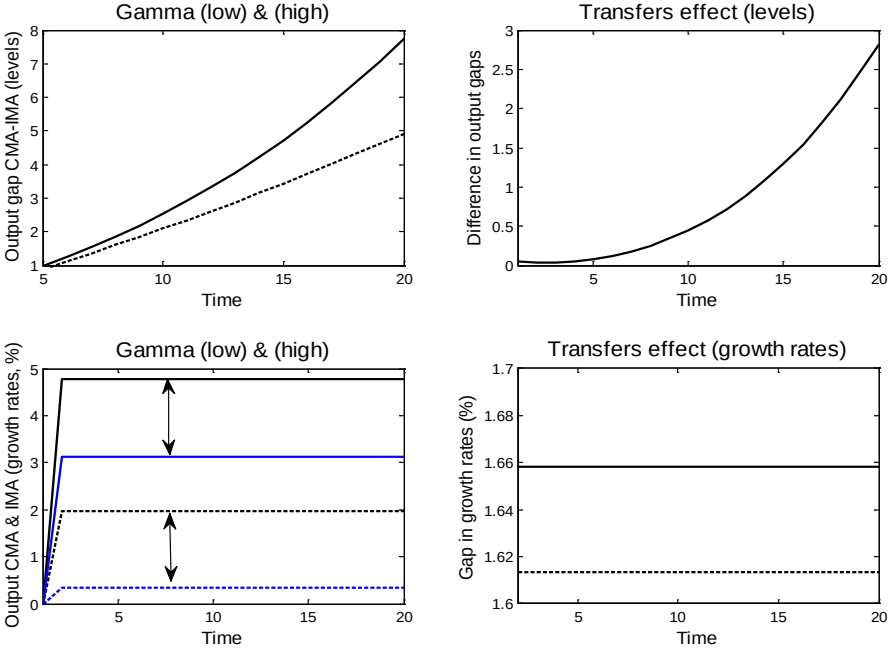


The constant income growth rate for the CMA allocation jumped to $g^{\star}$=0.0479, whilst for the IMA it increased to $g^{\#}$=0.0313. The intuition for the mechanisms driving this result goes as follows. A higher resource regeneration rate γ implied more abundant resources and higher rates of return on resource wealth. Now, for the CMA allocation the latter determined larger transfers, as can be seen in (12) and

(13). Larger intergenerational transfers imply stronger human capital accumulation which, combined with more abundant resources, boosted economic growth as summarized by the analytical formulation in (23). On the other hand, higher growth rate for the IMA was simply determined by more abundant resources as shown in (34). This unbalanced impact on growth rates implies that the gap in growth performances between the CMA and IMA allocations induced by the transfers has increased with the new regeneration rate γ_{high}.

In order to visualize this result, Figure 3 plots (in levels in the upper-left plot and in growth rates in the lower-left plot) the output gaps between allocations for different levels of γ (dashed lines for γ_{low}, solid for γ_{high}); jointly with the "Transfers effect" series (in levels in the upper-right plot and in growth rates in the lower-right plot) which show the period-by-period difference between them:

Figure 3



The series in Figure 3 show a positive effect of intergenerational transfers on the output gaps between allocations (both in levels and in growth rates) in response to a higher resource regeneration rate γ. In conclusion, Figure 3 and this subsection

more generally have shown that:

Remark 4 *Other things being equal, a higher resource regeneration rate γ boosts the positive effect of transfers on economic growth for both allocations, although it also implies a higher gap in growth performances between them.*

Interestingly, it can also be investigated whether intergenerational transfers increased as well as a fraction of total output. This is precisely one of the objectives of the next section in which financing of the transfers scheme is analyzed.

5.3 The revenue side: financing transfers

Let us now go into more detail about how the intergenerational transfers scheme actually works. In a parallel way to the theoretical result obtained by Boldrin and Montes (2005), I will show that the CMA equilibrium and efficiency can be fully implemented by the planner through a period-by-period scheme of intergenerational transfers financed through a lump-sum tax.

In each period t, a lump-sum tax τ_t is levied on adults and revenues $\Pi_t(\tau_t)$ are subsequently utilized to finance pensions to the old and education to the young. On the expenditure side, define $Z_t(z_t^P, z_t^E)$ as the total expenditure for transfers respectively of (PAYG) pensions z_t^P and education z_t^E. Consider the following government budget, balanced at any point in time:

$$\Pi_t(\tau_t) = Z_t(z_t^E, z_t^P), \qquad \forall t, \tag{45}$$

$$N_t^a \cdot \tau_t = N_t^y \cdot z_t^E + N_t^o \cdot z_t^P. \tag{46}$$

The budget constraints for the representative agent born in period $t-1$ would then read:

$$e_{t-1} = b_{t-1}, \tag{47}$$

$$c_t = w_t h_t - s_t - \left[z_t^E + z_t^P\right], \tag{48}$$

$$d_{t+1} = (1 + i_{t+1})\left(s_t + z_t^P\right). \tag{49}$$

Let us now compare (47-49) with the constraints faced by the representative member in the CMA allocation (12-14) and observe that, as long as pensions z_t^P and education transfers z_t^E are defined exactly as follows:

$$z_t^P = b_t(1+n), \qquad z_t^E = e_{t-1}^{\star}\left(1 + i^{\star}\right). \tag{50}$$

then the competitive equilibrium under the transfer policy achieves again the efficiency level of the CMA allocation.

Once the feasibility of introducing intergenerational transfers financed through a lump-sum tax has been assured, let us now investigate whether and how resource regeneration rate interacts with the transfer scheme (50). Recalling from the model calibration in the previous subsections that the growth of population has been set at the lower limit $n = 0$, let us therefore further assume without loss of generality that the mass of young, adult and old individuals is given by $N_t^y = N_t^a = N_t^o = 1$. This implies that (46) simplifies to $\tau_t = z_t^P + z_t^E = b_t + e_{t-1}^\star \left(1 + i^\star\right)$. By dividing both sides for $y_t^\star$ we obtain:

$$\frac{\tau_t}{y_t^\star} = \frac{b_t}{y_t^\star} + \frac{e_{t-1}^\star}{y_t^\star}\left(1 + i^\star\right) = \varepsilon^\star + \frac{\varepsilon^\star \cdot y_{t-1}^\star}{y_t^\star}\left(1 + i^\star\right) = \varepsilon^\star \left[1 + \frac{y_{t-1}^\star}{y_t^\star}\left(1 + i^\star\right)\right], \tag{51}$$

Now recall (20) and (23); inserting back into $\frac{\tau_t}{y_t^\star}$ provides:

$$\frac{\tau_t}{y_t^\star} = \varepsilon^\star \left(1 + \frac{\alpha\mu}{1 + \mu\varepsilon^\star}\right). \tag{52}$$

This result can be summarized as follows:

Remark 5 *The intergenerational transfer scheme given by (50) can be financed through a constant lump-sum tax relative to the output level* $\frac{\tau_t}{y_t^\star}$*, as shown in (52).*

In other words and for the case of transfers financed through lump-sum tax, higher resource regeneration rate does not influence the relative fraction of output devoted to their financing. To what extent was this result expected? Recall (18) in which the propensity to invest in human capital with respect to income was found *not* to be a function of the equilibrium interest rate. Now, the simulation of the previous subsection showed that the resource regeneration rate influences the levels of transfers only through its effect on the equilibrium interest rate. It is therefore straightforward to logically infer that (at least for the current case of lump-sum taxation) the propensity to invest in human capital (18) in respect to output does not vary in response to the resource regeneration rate γ.

6. Concluding remarks

This paper presented a model of a closed economy with overlapping generations of selfish agents, natural resources and human capital externalities. Resources are assumed to be necessary for production of consumption goods, they are initially owned by old agents and further transmitted to the adult generation via a standard competitive market of resource assets. Intuitively, natural resources differ from physical capital as a factor of production since they are gradually depleted rather than being accumulated through investment.

The question becomes then how to sustain economic growth in the longer run. Although natural resources that are essential for production are scarce, growth can be sustained through the accumulation of human capital that spills over across generations in the form of public knowledge. The crucial point is that the knowledge stock only grows if old agents invest in the education of the younger generation. This mechanism creates potential gains for intergenerational transfers independently of the problem of depleting resources. An intergenerational transfer scheme financing education for the young and pensions for the old is therefore implemented voluntarily by agents in the competitive economy with complete markets. The first step was to analytically pin down the properties of this bench-mark framework.

I proceeded by supposing that credit markets for education financing are instead missing or incomplete: young agents are no longer able to finance their education. Consequently, intergenerational transfers do not take place and the economy moves to an inefficient equilibrium. This framework was labeled as the IMA and was described in detail. The initial result was to quantify the substantial positive effects of transfers on economic growth. This was done by comparing (for a given level of the resource regeneration rate) the complete markets allocation with intergenerational transfers to the incomplete markets allocation without transfers.

The core contribution of the paper was then to investigate whether and how the degree of resource renewability influences intergenerational transfers. The qualitative intuition behind this question goes as follows. A higher resource regeneration rate implies that one of the two productive factors of the economy becomes more abundant. How does this variation impact the features of transfers, their effects on the growth performances between allocations and ultimately the fraction of output needed to finance them? The results showed that a higher resource regeneration rate γ expands the positive effects of transfers on growth performances

for both allocations but also on the gap in output growth rates between them. In addition, it is shown that transfers are financed at any point in time through a constant lump-sum tax relative to the output level. Notwithstanding their beneficial effects on economic growth, this means that whenever the degree of resource renewability varies, financing transfers does not employ a higher/lower fraction of output.

Let us now outline some possible directions for future research in this area. At first, considering different and more realistic taxation instruments to finance transfers might increase the variety and robustness of results for the benchmark framework developed in the current paper. Another natural step forward would be to ask what is the other side of the story. In other words, to look at how intergenerational transfers between generations of selfish agents affect the speed of resource depletion. Using a somewhat different framework from that of the current paper, Valente (2008) analyzed this problem in an overlapping generations model in which natural capital is owned by selfish old agents. He shows that transfers from old to young agents have the effects of increasing growth for all generations (except that of the first resource owner) and reducing the rate of resource depletion, hence preserving sustainability. A thorough theoretical analysis of these and related aspects combined with an empirical application regarding specific resource-rich economies would qualify as an interesting complementary study to the current paper.

MATHEMATICAL APPENDIX

1. No-arbitrage and Hotelling rule

1.1 No-arbitrage conditions and Hotelling rule: derivation of (10). Starting from expression (9), substitute the physical resource constraint (4) as $X_{t+1} = R_{t+1} - A_{t+1}$ to obtain

$$\frac{1}{N_t^a} \cdot \left(\frac{q_{t+1}A_{t+1} - p_{t+1}A_{t+1}}{1+i_{t+1}} + \frac{p_{t+1}R_{t+1}}{1+i_{t+1}} - q_t A_t \right).$$

Combining (4) with (3), substitute $R_{t+1}=A_t\ (1+\gamma)$ in the above expression to obtain

$$\frac{1}{N_t^a} \cdot \left(\frac{q_{t+1}A_{t+1} - p_{t+1}A_{t+1}}{1+i_{t+1}} + \frac{1+\gamma}{1+i_{t+1}} p_{t+1}A_t - q_t A_t \right). \quad \textbf{(53)}$$

Maximizing (53) with respect to A_{t+1} yields the static no-arbitrage condition $q_{t+1} = p_{t+1}$. Maximizing (53) with respect to A_t and substituting $p_{t+1} = q_{t+1}$ yields the generalized Hotelling rule (10) in the text.

2. Complete markets: full equilibrium characterization

2.1 Consumer Problem. Under complete credit markets, the typical young agent born in period $t-1$ maximizes utility (11) subject to (12, 13, 14) taking h_{t-1} and y_{t-1} as given and using $\left(c_t, d_{t+1}, s_t, b_{t-1}\right)$ as control variables. The problem can be simplified as follows. Substituting h_t in (12) by means of (14) and using (13) to eliminate s_t from the resulting expression yields the lifetime budget constraint

$$c_t + \frac{d_{t+1}}{1+i_{t+1}} = w_t \eta\left(h_{t-1}, e_{t-1}, y_{t-1}\right) - e_{t-1}\left(1+i_t\right). \quad \textbf{(54)}$$

Maximizing (11) subject to (54) using $\left(c_t, d_{t+1}, e_{t-1}\right)$ as control variables, the Lagrangian reads

$$L \equiv v\left(c_t\right) + \beta \cdot v\left(d_{t+1}\right) + \lambda\left[w_t \eta\left(h_{t-1}, e_{t-1}, y_{t-1}\right) - e_{t-1}\left(1+i_t\right) - c_t - \frac{d_{t+1}}{1+i_{t+1}} \right],$$

and the first-order conditions $L_{c_t} = 0$, $L_{d_{t+1}} = 0$ and $L_{e_{t-1}} = 0$ yield equations (15) and (16) in the text.

2.2 Equilibrium characterization. I will proceed to demonstrate that, in the log-linear model, output, resource use and human capital all grow at constant growth rates in each period $t = \{0,1,2,\ldots\infty\}$. Given the assumed learning technology (8), the

partial derivative with respect to education reads

$$\frac{\partial \eta\left(h_{t-1}, e^{\star}_{t-1} / y_{t-1}\right)}{\partial e^{\star}_{t-1}} = \mu \cdot \frac{h_{t-1}}{y_{t-1}}. \tag{55}$$

Consequently, the utility-maximizing condition for education investment (16) implies

$$w^{\star}_{t} = \left(1 + i^{\star}_{t}\right) \frac{1}{\mu} \cdot \frac{y_{t-1}}{h_{t-1}}, \tag{56}$$

so that the growth rate of wages is

$$\frac{w^{\star}_{t+1}}{w^{\star}_{t}} = \frac{1 + i^{\star}_{t+1}}{1 + i^{\star}_{t}} \cdot \frac{y_{t}}{y_{t-1}} \cdot \frac{h_{t-1}}{h_{t}} = \frac{1 + i^{\star}_{t+1}}{1 + i^{\star}_{t}} \cdot \frac{Y^{\star}_{t}}{Y^{\star}_{t-1}} \cdot \frac{H^{\star}_{t-1}}{H^{\star}_{t}}. \tag{57}$$

Results (56) and (57) are the crucial relationships telling us that, under the assumptions of the log-linear model, the economy exhibits constant growth rates of output and inputs in each period, from time zero to infinity. First, consider resource use. From the profit-maximizing condition on resource use in (2), the Hotelling rule (10) can be written as

$$\frac{X^{\star}_{t+1}}{X^{\star}_{t}} = \frac{Y^{\star}_{t+1}}{Y^{\star}_{t}} \cdot \frac{1 + \gamma}{1 + i^{\star}_{t+1}}. \tag{58}$$

Similarly, from the profit-maximizing condition on human capital in (2), the growth rate of wages is

$$\frac{w^{\star}_{t+1}}{w^{\star}_{t}} = \frac{Y^{\star}_{t+1}}{Y^{\star}_{t}} \cdot \frac{H^{\star}_{t}}{H^{\star}_{t+1}}, \tag{59}$$

where we can substitute (57) to obtain

$$\frac{1}{1 + i^{\star}_{t}} \cdot \frac{Y^{\star}_{t}}{Y^{\star}_{t-1}} \cdot \frac{H^{\star}_{t-1}}{H^{\star}_{t}} = \frac{1}{1 + i^{\star}_{t+1}} \cdot \frac{Y^{\star}_{t+1}}{Y^{\star}_{t}} \cdot \frac{H^{\star}_{t}}{H^{\star}_{t+1}}. \tag{60}$$

Hence, defining the convenient variable

$$\phi_{t+1} \equiv \frac{1}{1 + i^{\star}_{t+1}} \cdot \frac{Y^{\star}_{t+1}}{Y^{\star}_{t}}, \tag{61}$$

we can re-write the growth rates of inputs (58) and (61) as

$$\frac{X^{\star}_{t+1}}{X^{\star}_{t}} = \left(1 + \gamma\right) \cdot \phi_{t+1}, \tag{62}$$

$$\frac{H^{\star}_{t+1}}{H^{\star}_{t}} = \frac{\phi_{t+1}}{\phi_{t}} \cdot \frac{H^{\star}_{t}}{H^{\star}_{t-1}}. \tag{63}$$

The crucial step to show that these growth rates are constant over time is to combine the utility-maximizing condition for education investment (16) with firms' demand for human capital – that is, the profit-maximizing condition on human capital in (2). In fact, combining $w^{\star}_{t} = \left(1 + i^{\star}_{t}\right)\frac{1}{\mu} \cdot \frac{y_{t-1}}{h_{t-1}}$ with $w^{\star}_{t} = \alpha \frac{Y^{\star}_{t}}{H^{\star}_{t}}$, and recalling that $\frac{y_{t-1}}{h_{t-1}} = \frac{Y_{t-1}}{H_{t-1}}$, we can eliminate the wage rate and rearrange terms to get

$$\frac{1}{\alpha\mu} \cdot \frac{H^{\star}_{t}}{H_{t-1}} = \frac{Y^{\star}_{t}}{Y_{t-1}} \cdot \frac{1}{1 + i^{\star}_{t}} = \phi_{t}. \tag{64}$$

Substituting this result into (63) we obtain

$$\frac{H^{\star}_{t+1}}{H^{\star}_{t}} = \alpha\mu \cdot \phi_{t+1}. \tag{65}$$

2.3 Equilibrium value of the interest rate. Now, the growth rate of output is by definition equal to

$$\frac{Y^{\star}_{t+1}}{Y^{\star}_{t}} = \left(\frac{H^{\star}_{t+1}}{H^{\star}_{t}}\right)^{\alpha} \left(\frac{X^{\star}_{t+1}}{X^{\star}_{t}}\right)^{1-\alpha}. \tag{66}$$

Starting from (66), let us substitute the growth rates of human capital and resource use by means of (65) and (62), to obtain an expression that only contains $\frac{Y^{\star}_{t+1}}{Y^{\star}_{t}}$ and ϕ_{t+1}:

$$\frac{Y^{\star}_{t+1}}{Y^{\star}_{t}} = \left[\alpha\mu \cdot \phi_{t+1}\right]^{\alpha} \left[\left(1 + \gamma\right) \cdot \phi_{t+1}\right]^{1-\alpha} = \left(\alpha\mu\right)^{\alpha} \left(1 + \gamma\right)^{1-\alpha} \phi_{t+1}. \tag{67}$$

Therefore, we can utilize (61) and solve for the interest factor:

$$\left(1 + i^{\star}\right) = \left(\alpha\mu\right)^{\alpha} \left(1 + \gamma\right)^{1-\alpha}. \tag{68}$$

Hence the interest rate factor $\left(1 + i^{\star}\right)$ turns out to be a constant weighted average of the regeneration rate and of the marginal impact of the propensity to spend on education. But then, since the interest rate factor is constant in every period, output, resource use and human capital will grow at constant rates in every period as well.

2.4 Equilibrium value of the propensity to invest in education. To find the equilib-

rium propensity to invest in education, we need to follow a few sub-steps. At first, I obtain the aggregate budget constraint of the economy (a), total consumption of adult agents (b), total investment in education (c) and total consumption of old agents (d):

(a) the aggregate budget constraint of the economy. Substituting $b_{t-1} = e^{\star}_{t-1}$ in each period as well as the definition of aggregate savings of the adult $N^a_t s_t = q_t A_t$ in the budget constraints (12) and (13), the CMA is characterized by

$$N^a_t c^{\star}_t = w^{\star}_t h^{\star}_t N^a_t - q^{\star}_t A^{\star}_t - N^a_t e^{\star}_{t-1}\left(1+i^{\star}\right) - N^a_t e^{\star}_t\left(1+n\right), \tag{69}$$

$$N^o_{t+1} d^{\star}_{t+1} = q^{\star}_t A^{\star}_t\left(1+i^{\star}\right) + N^o_{t+1} e^{\star}_t\left(1+n\right)\left(1+i^{\star}\right). \tag{70}$$

Notice that setting (70) at time t and using the Hotelling rule (10), we have

$$N^o_t d^{\star}_t = q^{\star}_t A^{\star}_t + p^{\star}_t X^{\star}_t + N^o_t e^{\star}_{t-1}\left(1+n\right)\left(1+i^{\star}\right). \tag{71}$$

Hence, summing the aggregate consumption levels of adult and old agents, we obtain

$$N^a_t c^{\star}_t + N^o_t d^{\star}_t = w^{\star}_t h^{\star}_t N^a_t + p^{\star}_t X^{\star}_t - N^a_t e^{\star}_t\left(1+n\right),$$

where we can substitute $w^{\star}_t h^{\star}_t N^a_t + p^{\star}_t X^{\star}_t = Y^{\star}_t$ in view of constant returns to scale, and write the aggregate expenditure constraint of the economy as

$$Y^{\star}_t = N^y_t e^{\star}_t + N^a_t c^{\star}_t + N^o_t d^{\star}_t. \tag{72}$$

(b) total consumption of adult agents. Re-arranging terms in (70) we obtain

$$q^{\star}_t A^{\star}_t + N^o_{t+1} e^{\star}_t\left(1+n\right) = \frac{N^o_{t+1} d^{\star}_{t+1}}{1+i^{\star}}, \tag{73}$$

which can be substituted in (69) to get

$$c^{\star}_t + \frac{d^{\star}_{t+1}}{1+i^{\star}} = w^{\star}_t h^{\star}_t - e^{\star}_{t-1}\left(1+i^{\star}\right). \tag{74}$$

In the log-linear model, the utility function $v(\cdot) = \ln(\cdot)$ implies that the Euler condition (15) reduces to

$$\frac{d^{\star}_{t+1}}{1+i^{\star}} = \beta c^{\star}_t. \tag{75}$$

Substituting (75) in (74) yields

$$c^{\star}_t = \frac{w^{\star}_t h^{\star}_t - e^{\star}_{t-1}\left(1+i^{\star}\right)}{1+\beta}. \tag{76}$$

Notice that, multiplying both sides of (56) by $h_t^\star$ and substituting the learning technology (8), we obtain

$$w_t^\star h_t^\star - e_{t-1}^\star\left(1+i^\star\right) = \frac{1}{\mu}\cdot\left(1+i^\star\right)\cdot y_{t-1}. \tag{77}$$

Substituting (77) in (76), we obtain

$$c_t^\star = \frac{1+i^\star}{\left(1+\beta\right)\cdot\mu}\cdot y_{t-1}. \tag{78}$$

Multiplying both sides of (78) by $N_t^a = N_{t-1}^y$ we obtain the total consumption of adult agents

$$N_t^a c_t^\star = \frac{1+i^\star}{\left(1+\beta\right)\cdot\mu}\cdot Y_{t-1}^\star. \tag{79}$$

(c) total investment in education. From the learning technology (8), we have

$$\frac{e_t^\star}{y_t^\star} = \frac{1}{\mu}\cdot\frac{h_{t+1}^\star - h_t^\star}{h_t^\star}, \tag{80}$$

so that, multiplying both sides of (80) by N_t^y, we obtain the economy's total expenditure in education as

$$N_t^y e_t^\star = \frac{1}{\mu}\cdot\frac{h_{t+1}^\star - h_t^\star}{h_t^\star}\cdot Y_t^\star. \tag{81}$$

(d) total consumption of old agents. From (75) we have

$$d_t^\star = c_{t-1}^\star\cdot\beta\left(1+i^\star\right),$$

$$N_t^o d_t^\star = N_{t-1}^a c_{t-1}^\star\cdot\beta\left(1+i^\star\right),$$

where we can substitute equation (79) to obtain total consumption of old agents

$$N_t^o d_t^\star = \frac{\beta\left(1+i^\star\right)^2}{\mu\left(1+\beta\right)}\cdot Y_{t-2}^\star. \tag{82}$$

The next step: substitute total consumption of adult agents (b), total investment in education (c) and total consumption of old agents (d) into (a) and divide by output, obtaining propensities and therefore an expression linking output growth to human capital growth. Substitute (79), (81) and (82) into the aggregate constraint (72) obtaining

$$Y_t^\star = N_t^y e_t^\star + N_t^a c_t^\star + N_t^o d_t^\star, \tag{83}$$

$$Y_t^\star = \frac{1}{\mu}\cdot\frac{h_{t+1}^\star - h_t^\star}{h_t^\star}\cdot Y_t^\star + \frac{1+i^\star}{(1+\beta)\cdot\mu}\cdot Y_{t-1}^\star + \frac{\beta\left(1+i^\star\right)^2}{\mu(1+\beta)}\cdot Y_{t-2}^\star, \tag{84}$$

which, recalling the definition $\phi_t = \frac{1}{1+i^\star}\cdot\frac{Y_t^\star}{Y_{t-1}^\star}$, can be rearranged as

$$\frac{h_{t+1}^\star}{h_t^\star} = 1+\mu - \frac{1}{1+\beta}\cdot\frac{1}{\phi_t} - \frac{\beta}{1+\beta}\cdot\frac{1}{\phi_{t-1}}\cdot\frac{1}{\phi_t}. \tag{85}$$

Because $\frac{H_{t+1}^\star}{H_t^\star} = \alpha\mu\cdot\phi_{t+1}$ by (65), I can substitute

$$\phi_t = \frac{h_t^\star}{h_{t-1}^\star}\cdot\frac{1+n}{\alpha\mu} \text{ and } \phi_{t-1} = \frac{h_{t-1}^\star}{h_{t-2}^\star}\cdot\frac{1+n}{\alpha\mu}, \tag{86}$$

to obtain

$$\frac{h_{t+1}^\star}{h_t^\star} = 1+\mu - \frac{\alpha\mu}{(1+\beta)(1+n)}\cdot\frac{1}{\frac{h_t^\star}{h_{t-1}^\star}} - \frac{(\alpha\mu)^2\beta}{(1+\beta)(1+n)^2}\cdot\frac{1}{\frac{h_{t-1}^\star}{h_{t-2}^\star}}\cdot\frac{1}{\frac{h_t^\star}{h_{t-1}^\star}}. \tag{87}$$

Recall that the learning technology (8) defines $\frac{h_{t+1}^\star}{h_t^\star}$ as a function of the propensity to spend in education,

$$\frac{h_{t+1}^\star}{h_t^\star} = 1+\mu\varepsilon_t \equiv \Lambda(\varepsilon_t). \tag{88}$$

As a consequence, rewrite (87) as

$$\Lambda(\varepsilon_t) = 1+\mu - \frac{\alpha\mu}{(1+\beta)(1+n)}\cdot\frac{1}{\Lambda(\varepsilon_{t-1})} - \frac{(\alpha\mu)^2\beta}{(1+\beta)(1+n)^2}\cdot\frac{1}{\Lambda(\varepsilon_{t-1})}\cdot\frac{1}{\Lambda(\varepsilon_{t-2})}. \tag{89}$$

Equation (89) exhibits a steady state $\varepsilon^\star$ determined by the stationarity condition

$$\Lambda(\varepsilon^\star) = 1+\mu - \frac{\alpha\mu}{(1+\beta)(1+n)}\cdot\frac{1}{\Lambda(\varepsilon^\star)} - \frac{(\alpha\mu)^2\beta}{(1+\beta)(1+n)^2}\cdot\frac{1}{\Lambda(\varepsilon^\star)^2}. \tag{90}$$

This steady state is unstable:[8] if we start from $\varepsilon_0 \neq \varepsilon^\star$ at time zero, the dynamic

8. Importantly, note that the fact that equation (89) is an unstable (second-order) difference equation is good news: it implies that there exists one and only one value of $\varepsilon_t = \varepsilon^\star$ that is consistent with a bounded propensity to invest in education $0<\varepsilon_t<1$ in each $t = \{0,1,2,\ldots\infty\}$. If (89) were stable around some long-run steady state $\lim_{t\to\infty}\varepsilon_t = \tilde{\varepsilon} \in (0,1)$, agents could choose any initial value ε_0 at time zero, and then let the subsequent values ε_1, ε_2, assume the values dictated by equation (89), to end up with the long-run propensity $\tilde{\varepsilon} \in (0,1)$. But then, the whole

equation brings all subsequent values of ε_t away from the steady state $\varepsilon^\star$, implying

$$\text{either } \varepsilon_t = 0 \text{ or } \varepsilon_t = 1 \text{ at some finite time } t. \quad \textbf{(91)}$$

Is (91) possible in equilibrium? No. Given that I have shown that the interest rate is constant in each period, from time zero to infinity, it must be the case that the propensity to invest in education is bounded between zero and unity in each period t, from zero to infinity:

$$0< \varepsilon_t <1 \quad \text{in each} \quad t=\{0,1,2,\ldots\infty\} \quad \textbf{(92)}$$

The above condition must be true because, if ε_t diverges to zero or unity in finite time, there is no equilibrium interest rate in the economy (there is no human capital accumulation or there is no demand for human capital and resources). Given that (92) must be true, the only case in which we can satisfy equation (92) with $0< \varepsilon_t <1$ in each $t=\{0,1,2,\ldots\infty\}$ is that ε_t jumps at the steady state level $\varepsilon^\star$ at time zero, and remains constant thereafter, thereby satisfying equation (89) by being stuck in the steady state (90).

Remark 6 *The propensity to invest in education is constant and equal to*

$$\varepsilon_t = \varepsilon^\star \quad \text{in each } t=\{0,1,2,...\infty\},$$

where $\varepsilon^\star$ is determined by equation (90).

We can now derive the equilibrium growth rate of knowledge and, therefore, all the rest, as a function of $\varepsilon^\star$.

2.5 Equilibrium growth rates and propensions to consume. Given the above result in Remark 6, we can calculate the growth rates of inputs and output as a function of parameters and of $\varepsilon^\star$:

$$\frac{H^\star_{t+1}}{H^\star_t} = \frac{h^\star_{t+1}\ell_{t+1}}{h^\star_t \ell_t} = (1+n)\frac{h^\star_{t+1}}{h^\star_t} = (1+n)(1+\mu\varepsilon^\star). \quad \textbf{(93)}$$

Inserting this into (64) allows us to derive the constant growth rate of output:

$$\frac{Y^\star_{t+1}}{Y^\star_t} = \left(\frac{1+\gamma}{\alpha\mu}\right)^{1-\alpha}(1+n)(1+\mu\varepsilon^\star). \quad \textbf{(94)}$$

In conclusion, it is easy to substitute this result into (61) and in turn (62) to obtain the growth rate of resource use:

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equilibrium path is indeterminate because agents could start from any different ε_0 and end up in the same place $\tilde{\varepsilon} \in (0,1)$.

$$\frac{X^{\star}_{t+1}}{X^{\star}_{t}} = \frac{1+\gamma}{\alpha\mu}\left(1+n\right)\left(1+\mu\varepsilon^{\star}\right). \tag{95}$$

In order to fully characterize the CMA equilibrium we need also to calculate the propensities to consume of adult and old agents as functions of parameters and of $\varepsilon^{\star}$. Using the previous steps and recalling the aggregate constraint (72), I can calculate residually:

$$\frac{N^{a}_{t}c^{\star}_{t}}{Y^{\star}_{t}} = \frac{\alpha}{(1+\beta)\left(1+n\right)\left(1+\mu\varepsilon^{\star}\right)}, \tag{96}$$

$$\frac{N^{o}_{t}d^{\star}_{t}}{Y^{\star}_{t}} = \frac{\beta\alpha}{(1+\beta)\left(1+n\right)^{2}\left(1+\mu\varepsilon^{\star}\right)^{2}}. \tag{97}$$

3. Incomplete markets: full equilibrium characterization

3.1 Consumer Problem. This consumer problem will of course look like a simplified version of the above one. The typical young agent born in period $t-1$ will now maximize utility (11) subject to (24), (25) and (26) taking h_{t-1} as given and using $\left(c_{t}, d_{t+1}, s_{t}\right)$ as control variables. The problem can again be simplified as follows. Substituting h_{t} in (24) by means of (26) and using (25) to eliminate s_{t} from the resulting expression yields the lifetime budget constraint:

$$c_{t} + \frac{d_{t+1}}{1+i_{t+1}} = w_{t}h_{t-1}. \tag{98}$$

Maximizing (11) subject to (98) using $\left(c_{t}, d_{t+1}\right)$ as control variables, the Lagrangian reads

$$L \equiv v\left(c_{t}\right) + \beta\cdot v\left(d_{t+1}\right) + \lambda\left[w_{t}h_{t-1} - c_{t} - \frac{d_{t+1}}{1+i_{t+1}}\right],$$

and the first-order conditions $L_{c_{t}} = 0$ and $L_{d_{t+1}} = 0$ yield equation (27) in the above text.

3.2 Equilibrium characterization. As regards the aggregate budget of the economy, the allocation with incomplete markets IMA is characterized by

$$N^{a}_{t}c^{\#}_{t} = w^{\#}_{t}h^{\#}_{t}N^{a}_{t} - q^{\#}_{t}A^{\#}_{t}, \tag{99}$$

$$N^{o}_{t+1}d_{t+1} = q^{\#}_{t}A^{\#}_{t}\left(1+i^{\#}_{t+1}\right). \tag{100}$$

Notice that setting (100) at time t and using the Hotelling rule (10), we have

$$N_t^o d_t^\# = q_t^\# A_t^\# + p_t^\# X_t^\#. \tag{101}$$

Hence, summing the aggregate consumption levels of adult and old agents, we obtain

$$N_t^a c_t^\# + N_t^o d_t^\# = w_t^\# h_t^\# N_t^a + p_t^\# X_t^\#, \tag{102}$$

where we can substitute $w_t^\# h_t^\# N_t^a + p_t^\# X_t^\# = Y_t^\#$ in view of constant returns to scale, and write the aggregate expenditure constraint of the economy as

$$Y_t^\# = N_t^a c_t^\# + N_t^o d_t^\#. \tag{103}$$

Total consumption of adult agents. Re-arranging terms in (100) we obtain

$$q_t^\# A_t^\# = \frac{N_{t+1}^o d_{t+1}^\#}{1+i_{t+1}^\#}, \tag{104}$$

which can be substituted in (99) to get

$$c_t^\# + \frac{d_{t+1}^\#}{1+i_{t+1}^\#} = w_t^\# h_t^\#. \tag{105}$$

In the log-linear model, the utility function $v(\cdot)=\ln(\cdot)$ implies that the Euler condition (27) reduces to

$$\frac{d_{t+1}^\#}{1+i_{t+1}^\#} = \beta c_t^\#, \tag{106}$$

Substituting (106) in (105) yields

$$c_t^\# = \frac{w_t^\# h_t^\#}{1+\beta}. \tag{107}$$

Notice that from (2), we obtain

$$w_t^\# h_t^\# = \frac{\alpha Y_t^\#}{\ell_t}. \tag{108}$$

Substituting (108) in (107), we obtain

$$c_t^\# = \frac{\alpha}{(1+\beta)} \frac{Y_t^\#}{\ell_t}. \tag{109}$$

Multiplying both sides of (109) by $N_t^a = \ell_t = N_{t-1}^y$ and rearranging we obtain the propensity to consume of adult agents

$$\frac{N_t^a c_t^\#}{Y_t^\#} = \frac{\alpha}{(1+\beta)}. \tag{110}$$

Now the propensity to consume of old agents. By merging (103) and (109) we

have

$$Y_t^{\#} = \frac{\alpha N_t^a}{(1+\beta)} \frac{Y_t^{\#}}{\ell_t} + N_t^o d_t^{\#},$$

$$Y_t^{\#}\left(1 - \frac{\alpha}{1+\beta}\right) = N_t^o d_t^{\#},$$

$$\frac{N_t^o d_t^{\#}}{Y_t^{\#}} = \frac{1+\beta-\alpha}{1+\beta}. \quad \textbf{(111)}$$

Set now (111) at t+1 and solving for $d_{t+1}^{\#}$ gives:

$$d_{t+1}^{\#} = \left(\frac{1+\beta-\alpha}{1+\beta}\right) \frac{Y_{t+1}^{\#}}{N_{t+1}^o}. \quad \textbf{(112)}$$

Inserting back again this result and (109) into (106) gives:

$$1 + i_{t+1}^{\#} = \left(\frac{1+\beta-\alpha}{1+\beta}\right) \frac{Y_{t+1}^{\#}}{N_{t+1}^o} \frac{1}{\beta} \frac{(1+\beta)}{\alpha} \frac{N_t^a}{Y_t^{\#}}, \quad \textbf{(113)}$$

$$1 + i_{t+1}^{\#} = \left(\frac{1+\beta-\alpha}{\alpha\beta}\right) \frac{Y_{t+1}^{\#}}{Y_t^{\#}}. \quad \textbf{(114)}$$

Keep this result in mind as I move to resource use and output growth rate. Similarly as for the CMA allocation, from the profit-maximizing condition on resource use in (2), the Hotelling rule (10) can be rewritten as

$$\frac{X_{t+1}^{\#}}{X_t^{\#}} = \frac{Y_{t+1}^{\#}}{Y_t^{\#}} \cdot \frac{1+\gamma}{1+i_{t+1}^{\#}}. \quad \textbf{(115)}$$

Inserting (28) and (115) into the definition of the growth rate of output implies the following:

$$\frac{Y_{t+1}^{\#}}{Y_t^{\#}} = (1+n)\left(\frac{1+\gamma}{1+i_{t+1}^{\#}}\right)^{\frac{1-\alpha}{\alpha}}. \quad \textbf{(116)}$$

The growth rate of output $\frac{Y_{t+1}^{\#}}{Y_t^{\#}}$ appears to depend crucially on the ratio between resource regeneration and the interest rate. For levels of γ such that $\frac{1+\gamma}{1+i_{t+1}} < 1$, the growth rate of output might decrease. We are now just a few steps away from the final determination of the equilibrium interest rate and of the growth rates of the IMA. Recall (116) and substitute it into (114) to get:

$$1+i^{\#}=\left[\frac{(1+\beta-\alpha)(1+n)}{\alpha\beta}\right]^{\alpha}(1+\gamma)^{1-\alpha}. \tag{117}$$

Insert now (117) into (116) to get a final expression for the growth rate of the economy:

$$\frac{Y_{t+1}^{\#}}{Y_t^{\#}}=(1+n)^{\alpha}\left[\frac{\alpha\beta(1+\gamma)}{1+\beta-\alpha}\right]^{1-\alpha}. \tag{118}$$

In conclusion, from the Hotelling rule rewritten as in (115) we obtain the resource use growth rate:

$$\frac{X_{t+1}^{\#}}{X_t^{\#}}=\frac{\alpha\beta}{1+\beta-\alpha}(1+\gamma). \tag{119}$$

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