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THE DYNAMICS OF WEALTH, ENVIRONMENT AND LAND VALUE IN A THREE-SECTOR GROWTH MODEL

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Abstract
This study builds a three-sector growth model with endogenous wealth accumulation and environmental change. It deals with dynamics of land value and rent in tandem with economic growth and economic structural change. The economy consists of industrial, agricultural and environmental sectors. The land is used by the agricultural sector and for providing housing services. The model is based on the neoclassical growth theory and Ricardian theory. The household's decision is modeled with an alternative approach proposed by Zhang (1993). After building the model, we simulate the motion of the economy. We demonstrate that the economic system has a unique, stable, steady state. We also conduct comparative dynamic analysis. We demonstrate how exogenous changes in the environmental sector's productivity, the population, and the tax rates affect the consumption level of the industrial goods, the output level of the industrial sector, the rent income, the consumption of housing during transitory processes and in long-term equilibrium

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1. Introduction

Determination of land value over time and space is an important issue in free market economies. It makes common sense to argue that land prices vary over time and space in association with economic development. Nevertheless, only a few theoretical growth models include endogenous determination of land value in the economic growth theories with microeconomic foundation. This study makes a contribution to the literature of land value dynamics in a growth model with endogenous wealth accumulation and environmental change.

A well-known study of land rent is Ricardo's *On the Principles of Political Economy and Taxation* published in 1817. Ricardo studied income distribution to explain how a change in this distribution could hinder or favor accumulation. The Ricardian system attempts to interconnect wages, interest rate, and rent together in a compact theory. Ricardo distinguished between the three production factors, labor, capital, and land. He provided a theory to explain the functional income distribution of labor share, the capital, and the land rent share of total income. Ricardo (1821: preface) pointed out: "The produce... is divided among three classes of the commodity, namely, the proprietor of land, the owners of the stock or capital necessary for its cultivation, and laborers by whose industry it is cultivated. But in different stages of society, the proportions of the whole produce of the earth which will be allotted to each of these classes, under the names of rent, profits, and wages, will be essentially different; depending mainly on the actual fertility of the soil, on the accumulation of capital and population, and on the skill, ingenuity, and the instruments in agriculture." Since the publication of the *Principles*, many attempts have been done to extend or generalize the system (Barkai, 1959, 1966; Pasinetti, 1960, 1974; Cochrane, 1970; Brems, 1970; Caravale and Tosato, 1980; Casarosa, 1985; Negish, 1989; Morishima, 1989). Nevertheless, we can still apply what Ricardo (1821: preface) observed a long time ago to describe the current situation: "To determine the laws which regulate this distribution, is the principal problem in Political Economy: much as the science has been improved by the writings of Turgot, Stuart, Smith, Say, Sismondi, and others, they afford very little satisfactory information respecting the natural course of rent, profit, and wages." In Ricardo's statement there is no reference to land value (price). A genuine analysis of "the natural course" involves high-dimensional dynamic analysis as determination of land value and rent is related to many factors. As Cho (1996: 145) stated, "During the past decade, the number of studies on intertemporal changes in house prices has increased rapidly because of wider availability of extensive micro-level data sets, improvements in modeling techniques, and expanded business applications." The literature on house and land prices has been increasingly expanding since then (e.g., Bryan and Colwell, 1982; Case and Quigley, 1991; Chinloy, 1992; Clapp and Giaccotto, 1994; Calhoun, 1995; Quigley, 1995; Capozza and Seguin, 1996; Alpanda, 2012; Alexander, 2013; Du and Peiser, 2014; Kok *et al*. 2014). Most of these studies are empirical. There are only a few formal growth models with endogenous land values. According to Liu *et al*. (2011: 1), “Although it is widely accepted that house prices could have an important influence on macroeconomic fluctuations, quantitative studies in a general equilibrium framework have been scant.” Since land value is related to physical wealth which can be accumulated through saving, we need microeconomic mechanism to determine land value and saving in an integrated framework. The traditional Ricardian theory does not provide a proper microeconomic mechanism of wealth accumulation. On the other hand, the neoclassical growth theory models endogenous wealth accumulation with a microeconomic foundation. We will integrate the neoclassical growth theory with the Ricardian theory of distribution to determine land value and land rent.

Another important dimension of economic development is the environment. The environment affects productivities of firms and the welfare of households. On the other hand, both consumption and production pollute the environment. To deal with environmental change it is necessary to develop a dynamic interdependence between consumption, production, and a government’s environmental policies. The dynamic relations between consumption and pollution have been extensively analyzed in the literature of formal economic analysis since the publication of the seminal papers by Ploude (1972) and Forster (1973). There are an increasing attention to environmental change in the literature of economic growth and development (e.g., Copeland and Taylor, 2004; Stern, 2004; Dasgupta *et al.*, 2006). Three main processes are taken into account for explaining the level of environmental pollution (e.g., Tsurumi and Managi, 2010). They are: (i) increases in output; (ii) changes in preferences or policy changes; and (iii) income changes (Pearson, 1994; Grossman, 1995; Brock and Taylor, 2006; and Kijima *et al.*, 2010). Another important phenomenon is the so-called environmental Kuznets curve (e.g., Kijima *et al.,* 2010). There is a large number of empirical studies on the environmental Kuznets curve for various pollutants. These studies identify different relations - for instance, inverted U-shaped relationship, a U-shaped relationship, monotonically increasing or monotonically decreasing relationship - between pollution and rising per capita income levels (e.g., Dinda, 2004; Managi, 2007; Tsurumi and Managi, 2010). The ambiguous or situation-dependent relations between environmental quality and economic growth and the inability of economic growth theory to explain properly these observed phenomena implies the necessity for developing more comprehensive theories.

This study builds an economic growth model with endogenous environment and dynamics of land value and land rent. The economic production and market aspects are based on the neoclassical growth theory. The models in the neoclassical growth theory are extensions and generalizations of the pioneering works of Solow (1956). As far as the economic structure is concerned, this study is influenced by Uzawa (1961), who made an extension of Solow’s one-sector economy by a breakdown of the productive system into two sectors using capital and labor (see also, Diamond, 1965; Stiglitz, 1967; Drugeon and Venditti, 2001; Zhang, 2005). In the Uzawa two-sector growth model, one sector produces industrial goods and the other consumption goods. We extend the Uzawa analytical framework to an economy with agricultural production and housing. It should be noted that we deviate from the traditional approaches in the neoclassical growth theory in that we analyze behavior of households with the alternative approach proposed by Zhang (1993). There are three main approaches to household behavior in economic growth theory with wealth accumulation. The Solow model is the starting point for almost all analyses of economic growth (Solow, 1956). The Solow model does not provide a mechanism of endogenous savings. Another important approach is the so-called representative agent growth model based on Ramsey's utility function (Ramsey, 1928). The Ramsey approach assumes that utility is additive over time. Cass and Koopmans integrated Ramsey's analysis of consumer optimization and Solow's description of profit-maximizing producers within a compact framework (Cass, 1965; Koopmans, 1965). A main problem of this approach is that it makes the analysis intractable even for a simple economic growth problem. Another approach in economic modeling is the so-called OLG approach. In his original contribution to growth theory with capital accumulation, Diamond (1965) used the overlapping generation's structure proposed by Samuelson (1958) to examine the long-term dynamical efficiency of competitive production economies. The approach is a discrete version of the continuous Ramsey approach. Most models of the approach assume that agents live only two periods – as mentioned in Azaridias (1993), each period should last over 30 years if one really wants to use analytical results to provide direct insights into reality. The time period of over 30 years period is generally considered too long for discussing modern economic changes because within each period nothing is allowed to be changeable. The model in this study is an integration of the two models recently proposed by Zhang (2014, 2015). This model is different from Zhang's 2014 model in that this study introduces endogenous environment and land value based on Zhang's 2015 model with environment. The rest of the paper is organized as follows. Section 2 defines the basic model. Section 3 shows how we solve the dynamics and simulates the model. Section 4 examines effects of changes in some parameters on the economic system over time. Section 5 concludes the study. The appendix proves the main results in Section 3.

2. The model

The economy consists of industrial, agricultural and environment sectors. The industrial sector produces goods, which are freely traded on the market. The industrial production is the same as that in Solow's one-sector neoclassical growth model. It is a commodity used both for investment and consumption. The agricultural sector produces agricultural goods, which are used for consumption. In our economic system there is an environmental sector which keeps the environment clean and is financially supported by the government. The government's income comes for taxing producers and consumers. The population N is homogenous and constant. We neglect issues related to the endogenous population and will study effects of exogenous population changes on economic structure and land values. The total land L is homogenous and constant. The land is owned by households and is distributed between housing and agricultural production in the competitive land market. The assumption of fixed land is strict. As observed by Glaeser, *et al.* (2005), the land supply elasticity varies substantially over space in the USA (see also, Davis and Heathcote, 2007). This study neglects possible changes in land supply. Households achieve the same utility level regardless of what profession they choose. All the markets are perfectly competitive. We select industrial goods to serve as numeraire.

The industrial sector

We assume that the production of the industrial sector is through combining the labor force, $N_i(t)$ and the physical capital, $K_i(t)$. We use the following conventional production function to describe a relationship between inputs and output

$$F_i(t) = A_i \Gamma_i(E(t)) K_i^{\alpha_i}(t) N_i^{\beta_i}(t), \quad A_i, \alpha_i, \beta_i > 0, \quad \alpha_i + \beta_i = 1, \tag{1}$$

where $\Gamma_i(E)$ is a function of the environmental quality measured by the level of pollution, $E(t)$, and A_i , α_i and β_i are parameters. The parameter A_i is the total factor productivity. The parameters α_i and β_i are the output elasticities of capital and labor, respectively. The productivity of the sector is negatively related to the pollution level, i.e., $d\Gamma_i / E \le 0$. The production function is a neoclassical one and homogeneous of degree one with the inputs. Markets are competitive; thus labor and capital earn their marginal products. The rate of interest $r(t)$ and wage rate $w(t)$ are determined by markets. The marginal conditions are given by

$$r(t) + \delta_k = \frac{\alpha_i \bar{\tau}_i F_i(t)}{K_i(t)}, \quad w(t) = \frac{\beta_i \bar{\tau}_i F_i(t)}{N_i(t)}, \tag{2}$$

where δ_κ is the fixed depreciation rate of physical capital and $\bar{\tau}_i \equiv 1 - \tau_i$ where τ_i is the fixed tax rate of the industrial sector's output, $0 < \tau_i < 1$.

The agricultural sector

We assume that agricultural production is the result of combining capital $K_\alpha(t)$ labor force $N_\alpha(t)$ and land $L_\alpha(t)$ as follows

$$F_a(t) = A_a \Gamma_a(E(t)) K_a^{\alpha_a}(t) N_a^{\beta_a}(t) L_a^{\varsigma}(t), \quad A_a, \alpha_a, \beta_a, \varsigma > 0, \quad \alpha_a + \beta_a + \varsigma = 1, \tag{3}$$

where $\Gamma_\alpha(E(t))$ is similarly defined as $\Gamma_i(E(t))$, and A_α, α_α, β_α and ς are parameters.

The marginal conditions are

$$r(t) + \delta_k = \frac{\alpha_a \bar{\tau}_a p_a(t) F_a(t)}{K_a(t)}, \quad w(t) = \frac{\beta_a \bar{\tau}_a p_a(t) F_a(t)}{N_a(t)}, \quad R(t) = \frac{\varsigma \bar{\tau}_a p_a(t) F_a(t)}{L_a(t)}, \tag{4}$$

where $p_\alpha(t)$ is the price of agricultural goods, $R(t)$ is the land rent, and $\bar{\tau}_a \equiv 1 - \tau_a$ where τ_α is the fixed tax rate on the agricultural sector's output, $0 < \tau_\alpha < 1$.

Choice between physical wealth and land

Land may be owned by different agents under various institutions. We assume that land is privately owned by households. There are different approaches with regard to determination of land prices and rents. For instance, in some approaches (Iacoviello, 2005; Iacoviello and Neri, 2010) households are assumed to be credit constrained and these households use land or houses as collateral to finance consumption expenditures. These models with credit-constrained households are used to explain positive co-movements between house prices and consumption expenditures (see also, Campbell and Mankiw, 1989; Zeldes, 1989; Case, *et al.*, 2005; Mian and Sufi, 2010; Oikarinen, 2014), even though they are not effective in explaining well-observed positive co-movements between land prices and business investment. In Liu *et al.* (2011), instead of households, firms are assumed to be credit constrained. Firms finance investment spending by using land as a collateral asset. We assume that land can be sold and bought in free markets without any friction and transaction costs. Land use will not waste land and land cannot regenerate itself. Households own land and physical wealth. We use $p_L(t)$ to denote the price of land. Consider now an investor with one unity of money. He can either invest in capital goods thereby earning a profit equal to the net own-rate of return $r(t)$ or invest in land thereby earning a profit equal to the net own-rate of return $R(t)/p_L(t)$. As we assume capital and land markets to be at competitive equilibrium at any point in time, two options must yield equal returns, i.e.

$$\frac{(1 - \tau_R) R(t)}{p_L(t)} = (1 - \tau_k) r(t), \tag{5}$$

where τ_R and τ_k are the fixed tax rates of the land rent income and income from interest payment on wealth, respectively. This equation enables us to determine the choice between owning land and wealth. This assumption is made under many strict conditions. For instance, we neglect any transaction costs and any time needed for buying and selling. The expectations of land are complicated. Equation (5) also implies perfect information and rational expectation.

Consumer behavior

For simplicity, we use the lot size to stand for housing. As argued, for instance, by Davis and Heathcote (2007), most of the fluctuations in house prices are driven by land price rather than by the cost of structures. Consumers decide consumption levels of industrial and agricultural goods and lot size, as well as on how much to save. The land distribution between the agricultural use and residential use is determined in competitive land markets. The land rent is equal both for the land used for agricultural production and for the residential use. This study uses the approach to consumers' behavior proposed by Zhang (1993). We denote physical wealth by $\bar{k}(t)$ and land $\bar{l}(t)$ owned by the representative household, respectively. The total value of wealth owned by the household $a(t)$ is the sum of the two assets

$$a(t) = \bar{k}(t) + p_L(t)\bar{l}(t). \tag{6}$$

Per capita current income from the interest payment $r(t)\bar{k}(t)$, the wage payment $w(t)$, and the land revenue $R(t)\bar{l}(t)$ is given by

$$y(t) = (1 - \tau_k)r(t)\bar{k}(t) + (1 - \tau_w)w(t) + (1 - \tau_R)R(t)\bar{l}(t), \tag{7}$$

where τ_k, τ_w, and τ_R are respectively the fixed tax rates on the wealth income, wage, and land rent income. It should be noted that in the Solow growth model $y(t)$ is the disposable income. In our approach we call $y(t)$ the current income. In our approach the disposable income is the sum of the current value and the value of the household's wealth. It is evident that what a household is is not only what the household currently earns, but also should include the value of the household's wealth (if the wealth could be sold instantaneously without any transaction costs). In this study we assume that agents can sell and buy their assets instantaneously without any transaction costs. This is obviously a strict assumption. This assumption simplifies modelling behavior of agents. The per capita disposable income is given by

$$\hat{y}(t) = y(t) + a(t). \tag{8}$$

At each point in time, a consumer would distribute the total available budget between the saving $s(t)$, the consumption of industrial goods $c_i(t)$, the consumption of agricultural goods $c_a(t)$, and the lot size $l_h(t)$. Let $\tilde{\tau}_c$, $\tilde{\tau}_a$, $\tilde{\tau}_h$ stand for, respectively, the fixed tax rates on consumption of industrial good, agricultural good, and housing. The budget constraint is given by

$$(1 + \tilde{\tau}_c)c_i(t) + s(t) + (1 + \tilde{\tau}_a)p_a(t)c_a(t) + (1 + \tilde{\tau}_h)R(t)l_h(t) = \hat{y}(t). \tag{9}$$

The left-hand side is the total expenditure on consumption and saving. The right-hand side is the value of the total available income. The representative household has four variables, $s(t)$, $c_i(t)$, $c_a(t)$, and $l_h(t)$, to decide. It should be remarked that $\bar{l}(t)$ and $l_h(t)$ represent the land owned by the representative household and the representative household's lot size, respectively. All the households and producers pay the same land rent. The consumer's utility function is specified as follows

$$U(t) = \Gamma_c(E(t))c_i^{\xi_0}(t)c_a^{\mu_0}(t)l_h^{\eta_0}(t)s^{\lambda_0}(t), \quad \xi_0, \mu_0, \eta_0, \lambda_0 > 0,$$

in which ξ_0, μ_0, η_0, and λ_0 are the household's elasticity of utility with regard to industrial goods, agricultural goods, housing, and saving. We call ξ_0, μ_0, η_0, and λ_0 propensities to consume industrial goods, agricultural goods, housing, and to hold wealth, respectively. Saving from the disposable income enables the household to accumulate wealth, increasing the household's welfare. It should be noted that this study does not explicitly take account of consumers' awareness of the environment. For instance, consumers may prefer environment-friendly goods when their living conditions are changed. We may take account of changes in consumers' behavior, for instance, by assuming that the representative consumer spends a proportion of the disposable income on environment or the tax rate on the consumer's consumption is explicitly related to income and consumption level.

Maximizing the utility $U(t)$ subject to (9) yields

$$c_i(t) = \xi\,\hat{y}(t), \quad p_a(t)c_a(t) = \mu\,\hat{y}(t), \quad R(t)l_h(t) = \eta\,\hat{y}(t), \quad s(t) = \lambda\,\hat{y}(t), \tag{10}$$

where

$$\xi \equiv \frac{\rho\,\xi_0}{1+\tilde{\tau}_c}, \quad \mu \equiv \frac{\rho\,\mu_0}{1+\tilde{\tau}_a}, \quad \eta \equiv \frac{\rho\,\eta_0}{1+\tilde{\tau}_h}, \quad \lambda \equiv \rho\,\lambda_0, \quad \rho \equiv \frac{1}{\xi_0+\mu_0+\eta_0+\lambda_0}.$$

Wealth accumulation

According to the definition of $s(t)$, the change in the household's wealth is given by

$$\dot{a}(t) = s(t) - a(t). \tag{11}$$

The equation simply states that the change in wealth is equal to saving minus dissaving.

Balances of demand and supply for agricultural goods

The demand and supply for the agricultural sector's output balance at any point in time

$$C_a(t) = c_a(t)N = F_a(t). \tag{12}$$

All the land owned by households

The land owned by the population is equal to the national available land

$$\bar{l}(t)N = L. \tag{13}$$

Full employment of capital

We use $K(t)$ to stand for the total capital stock. We assume that the capital stock is fully employed. We have

$$K_i(t) + K_a(t) + K_e(t) = K(t). \tag{14}$$

where $K_e(t)$ is the capital stocks employed by the environmental sector.

Balances of demand and supply for industrial goods

The demand and supply for the industrial sector's output balance at any point in time

$$\dot{K}(t) = F_i(t) - c_i(t)N - \delta_k K(t).$$

According to Say's law, we can consider this equation redundant in the general equilibrium system.

The value of physical wealth and capital

The value of physical capital is equal to the value of physical wealth

$$\bar{k}(t)N = K(t). \tag{15}$$

Full employment of the labor force

We assume that the labor force is fully employed

$$N_i(t) + N_a(t) + N_e(t) = N. \tag{16}$$

The land market clearing condition

The condition that land is fully used for the agricultural production and residential use implies

$$l_h(t)N + L_a(t) = L. \tag{17}$$

Dynamics of pollutants and the environment sector

We now describe dynamics of the stock of pollutants, $E(t)$. We assume that pollutants are created both by production and consumption. We specify the dynamics of the stock of pollutants as follows

$$\dot{E}(t) = \theta_i F_i(t) + \theta_a F_a(t) + \tilde{\theta}_i C_i(t) + \tilde{\theta}_a C_a(t) - Q_e(t) - \theta_0 E(t), \tag{18}$$

in which θ_i, θ_a, $\tilde{\theta}_i$, $\tilde{\theta}_a$, and θ_0 are positive parameters and

$$Q_e(t) = A_e \Gamma_e(E) K_e^{\alpha_e}(t) N_e^{\beta_e}(t), \quad A_e, \alpha_e, \beta_e > 0, \tag{19}$$

where $N_e(t)$ is the labor force employed by the environmental sector, A_e, α_e, and β_e are positive parameters, and $\Gamma_e(E)$ (≥ 0) is a function of E. As one referee points out, equation (18) implies end-of-pipe abatement technology, as opposed to process-integrated technology. The end-of-pipe abatement technology reduces the amount of contaminants before they form. The process-integrated technology is used after the production process to remove already formed contaminants (Palivos and Varvarigos, 2015). The term $\theta_i F_i$ (or $\theta_a F_a$) means that pollutants that are emitted during production processes are linearly positively proportional to the output level (Gutiérrez, 2008). The parameter, $\tilde{\theta}_i$ (or $\tilde{\theta}_a$) means that in consuming one unit of the good the quantity $\tilde{\theta}_i$ (or $\tilde{\theta}_a$) is left as waste (Prieur, 2009). The parameter $\tilde{\theta}_i$ depends on the technology and environmental sense of consumers. The parameter θ_0 is called the rate of natural purification. The term $\theta_0 E$ measures the rate that the nature purifies environment. The term, $K_e^{\alpha_e} N_e^{\beta_e}$, in Q_e means that the purification rate of the environment is positively related to capital and labor inputs. The function $\Gamma_e(E)$, implies that the purification efficiency is dependent on the stock of pollutants. For simplicity, we specify Γ_e as follows $\Gamma_e(E) = E^{b_e}$, $1 > b_e \geq 0$. This equation means that the productivity of the environment sector is positively related to the level of pollutants. The more the environment is polluted, the more pollutants the environment sector can clear per unit of time with the same inputs (in the case of $b_e > 0$). We require $1 > b_e$ as doubling the level of pollutants would not double the environment's productivity.

We now determine how the government determines the number of the labor force and the level of capital employed for purifying pollution. We assume that all the tax incomes are spent on the environment. In the literature of environmental economics, for instance, Ono (2003) introduces tax on the producer and uses the tax income for environmental improvement in the traditional neoclassical growth theory. Different from Ono's approach, our study takes account of tax not only on producers, but also on consumers. The government's tax incomes consist of the tax incomes on the production sectors, consumption, wage income and wealth income. Hence, the government's income is given by

$$Y_e(t) = \tau_i F_i(t) + \tau_a p_a(t) F_a(t) + I_c(t), \tag{20}$$

where

$$I_c(t) \equiv \left[\tilde{\tau}_c c_i(t) + \tilde{\tau}_a p_a(t) c_a(t) + \tilde{\tau}_h R(t) l_h(t) + \tau_k r(t) \bar{k}(t) + \tau_w w(t) + \tau_R R(t) \bar{l}(t)\right] N.$$

For simplicity, we assume that the government's income is used up only for the environmental purpose. As there are only two input factors in the environmental sector, the government budget is given by

$$\left(r(t)+\delta_k\right)K_e(t)+w(t)N_e(t)=Y_e(t). \tag{21}$$

We need an economic mechanism to determine how the government distributes the tax income. As in Zhang (2005), we assume that the government employs the labor force and capital stocks for purifying the environment in such a way that the purification rate achieves its maximum under the given budget constraint. The government's optimal problem is thus given by maximizing (19) under constraint (21). That is

$$\text{Max } Q_e(t) \quad \text{s.t. (21)}$$

The optimal solution is given by

$$\left(r(t)+\delta_k\right)K_e(t)=\alpha Y_e(t), \quad w(t)N_e(t)=\beta Y_e(t), \tag{22}$$

where

$$\alpha \equiv \frac{\alpha_e}{\alpha_e+\beta_e}, \quad \beta \equiv \frac{\beta_e}{\alpha_e+\beta_e}.$$

We thus built the model. The model describes dynamics of the economic structure and land value. We now examine dynamic properties of the model.

3. The dynamics and the motion by simulation

We built a model with endogenous wealth accumulation and environment. The model also determines the value of land and land rent. The system contains many variables which are connected to each other with nonlinear relations. It is impossible to give analytical solutions to this kind of nonlinear differential equations. For illustration, the rest of the study simulates the model. In the appendix, we show that the dynamics of the national economy can be expressed as two differential equations. First, we introduce a variable $z(t)$ by

$$z(t) \equiv \frac{r(t)+\delta_k}{w(t)}.$$

We now show that the dynamics can be expressed by two dimensional differential equations with $z(t)$ and $E(t)$ as the variables.

Lemma

The motion of the system is determined by the following two differential equations

$$\dot{z} = \Lambda(z, E),$$
$$\dot{E} = \Omega(z, E),$$

where the right-hand sides of (23) are functions of $z(t)$ and $E(t)$ determined in the appendix. Moreover, all the other variables can be determined as functions of $z(t)$ and $E(t)$ at any point in time by the following procedure: $r(t)$ and $w(t)$ by (A2) $\rightarrow \bar{k}_1(t)$ by (A28) $\rightarrow K_a(t)$ by (A6) $\rightarrow K_i(t)$ and $K_e(t)$ by (A19) $\rightarrow N_i(t)$, $N_a(t)$ and $N_e(t)$ by (A1) $\rightarrow \hat{y}(t)$ by (A13) $\rightarrow L_a$ and l_h by (A9) $\rightarrow \bar{l}$ by (13) $\rightarrow p_L(t)$ by (A21) $\rightarrow R(t)$ by (A12)$\rightarrow a(t)$ by (A23)$\rightarrow p_a(t)$ by (A5) $\rightarrow F_i(t)$ by (1) $\rightarrow F_a(t)$ by (3) $\rightarrow c_i(t)$, $c_a(t)$, and $s(t)$ by (5) $\rightarrow Y_e(t)$ by (22) $\rightarrow Q_e(t)$ by (19) $\rightarrow U(t)$ by the definition.

The lemma shows that once we determine the values of the two variables with initial conditions, we determine all the other variables in the economic system. The lemma is important as it gives a procedure to follow the motion of the system by computer. First we specify environment-related functions as follows:

$$\Gamma_m(t) = E^{-b_m}(t), \quad m = i, a, e, c.$$

In order to follow the motion of the system, we specify the parameter values as follows:

$$N = 20,\ L = 8,\ A_i = 1,\ A_a = 0.8,\ A_e = 0.5,\ \alpha_i = 0.32,\ \alpha_a = 0.1,\ \beta_a = 0.2,\ \alpha_e = 0.3,$$
$$\beta_e = 0.7,\ \lambda_0 = 0.7,\ \xi_0 = 0.15,\ \mu_0 = 0.06,\ \eta_0 = 0.06,\ b_i = b_a = 0.15,\ b_e = -0.1,\ b_c = 0.1,$$
$$\tau_m = 0.01,\ m = i, a, k, R, w,\ \tilde{\tau}_m = 0.01,\ m = h, a, c,\ \theta_i = 0.1,\ \theta_a = 0.05,\ \tilde{\theta}_i = 0.05,$$
$$\tilde{\theta}_a = 0.01,\ \theta_0 = 0.05,\ \delta_k = 0.05. \tag{24}$$

The population is fixed at 20 and the land at 8. We assume that the propensity to save is much higher than the propensity to consume industrial goods and the propensity to consume agricultural goods. The tax rates are approximately 1 percent. The environmental impact parameters b_m are about 0.1 to 0.15. Although these parameter values are not based on any real economy, we choose them for the purpose of simulation. We also need to specify initial conditions. As shown in the lemma, the differential equations contain the two variables, $z(t)$ and $E(t)$. We specify the following initial conditions:

$$z(0) = 0.5, \quad E(0) = 7.5.$$

As shown in the appendix, the following variables are invariant in time:

$$l_h = 0.236, \ \ L_a = 3.275, \ \ \bar{l} = 0.4.$$

We plot the motion of the other variables in Figure 1. In Figure 1 the national gross product (GDP) is

$$Y(t) = F_i(t) + p_a(t)F_a(t) + l_h N R(t).$$

It should be remarked that the paths of the changes of all the variables in the figure are dependent on the initial conditions specified. For the given initial conditions the GDP and national capital stock fall over time till they become stationary. The wage rate, the price of land, price of agricultural goods, and land rent are reduced, and the rate of interest is enhanced. The output levels of the agricultural and industrial sectors are reduced. Some of the force is shifted from the agricultural and environment sectors to the industrial sector. The capital inputs of the three sectors are increased. The physical wealth, total wealth, and consumption levels of the two goods are reduced. The output of the environment sector falls and environment is deteriorated. It should be noted that the dynamic relationship between the GDP and the land price plotted in Figure 1 is similar to the phenomenon described by Liu *et al*. (2011: 1): "The recent financial crisis caused by a collapse of the housing market propelled the U.S. economy into the Great Recession. A notable development during the crisis period was a slump in business investment in tandem with a sharp decline in land prices." The conclusions made by Liu *et al*. are based on the data for the Great Recession period as well as for the entire sample period from 1975 to 2010.

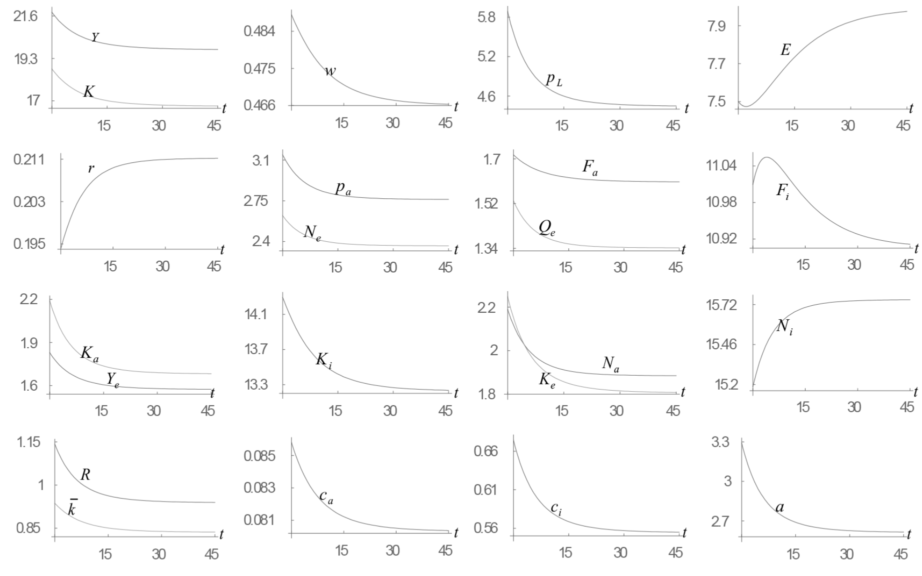

Figure 1. The Motion of the Economic System

From Figure 1 we observe that all the variables tend to become stationary in the long term. This implies the existence of some equilibrium point. We confirm the existence of a unique equilibrium point as follows

$$Y = 19.78,\ Y_e = 1.57,\ K = 16.71,\ E = 7.99,\ w = 0.47,\ p_L = 4.44,\ R = 0.94,\ r = 0.21,$$
$$p_a = 2.76,\ F_a = 1.61,\ F_i = 10.91,\ Q_e = 1.34,\ K_a = 1.68,\ K_i = 13.23,\ K_e = 1.81,\ N_a = 1.88,$$
$$N_i = 15.75,\ N_e = 2.36,\ L_a = 3.28,\ l_h = 0.24,\ \bar{l} = 0.4,\ \bar{k} = 0.84,\ c_a = 0.08,\ c_i = 0.55,$$

$$\alpha = 2.61.$$

The two eigenvalues at the equilibrium point are
–0.189, –0.093.

The confirmed stability is important as this guarantees that we can effectively conduct comparative dynamic analysis.

4. Comparative dynamic analysis

We now examine effects of changes in some parameters on the motion of the economic system. As the lemma gives a computational procedure to calibrate the motion of all the variables and the equilibrium point is locally stable, it is straightforward to conduct comparative dynamic analysis. In the rest of this study we use $\bar{\Delta} x_j(t)$ to mean the change rate of the variable, $x_j(t)$, in percentage due to changes in a parameter value.

A rise in the propensity to consume housing

First we examine the effects of the following change in the propensity to consume housing on the economic variables: $\eta_0 : 0.06 \Rightarrow 0.07$. As the household has a stronger preference for housing, the land is redistributed as follows

$$\bar{\Delta} L_a = -8.96,\ \bar{\Delta} l_h = 6.21,\ \bar{\Delta} \bar{l} = 0.$$

More land is relocated to residential use. The effects on the other variables are plotted in Figure 2. The rent of lot size, the land value, and rate of interest are increased. The wage rate falls initially and is reduced in the long term. The price of agricultural goods is increased. The household's physical wealth per capita is reduced. The household's wealth is enhanced. The household consumes less agricultural goods. Initially the household consumes more industrial goods and consumes slightly less agricultural goods in the long term. The national capital stock is reduced and the GDP is increased. The output of the industrial sector falls and the output of the agricultural sector is enhanced. The government receives more income and the environment sector's output is enhanced. In the long term some of the industrial sector's labor force is shifted to the environment sector. The environment is improved. The household gets higher utility.

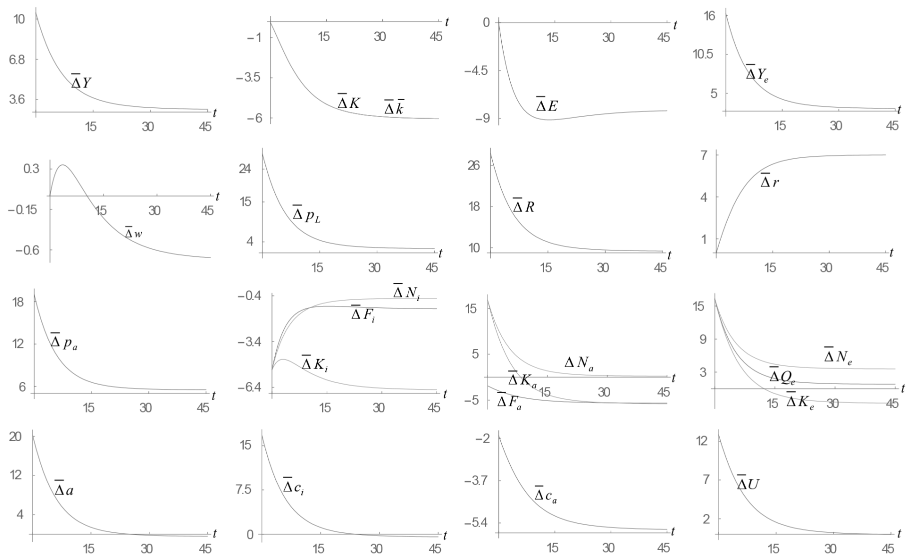

Figure 2. A Rise in the Propensity to Consume Housing

A rise in the propensity to consume industrial goods

We now allow the propensity to consume industrial goods to be increased as follows: $\xi_0 : 0.15 \Rightarrow 0.17.$ The land distribution is not affected by the preference change. The effects on the other variables are plotted in Figure 3. As the household spends more out of the disposable income on consuming industrial goods, the total capital stock and the GDP are reduced. The fall in the total capital stock is in tandem with the rise in the rate of interest and the fall in the wage rate. Both the land value and the land rent are reduced. The household holds less wealth and physical wealth. The household's consumption level of industrial goods rises initially and falls in the long term. This occurs as the income falls in the long term. The price of agricultural goods falls in association with falling output of the sector. The output level and the two inputs of the industrial sector are increased initially. In the long term the output level and capital of the industrial sector are reduced. The reallocation of the labor force occurs from the agricultural and environment sectors to the industrial sector. The capital inputs of all the sectors are reduced in the long term. The government's income falls in tandem with the fall in the output of the environment sector. The environment deteriorates. The household has less wealth, consumes less agricultural goods, and has a lower utility level.

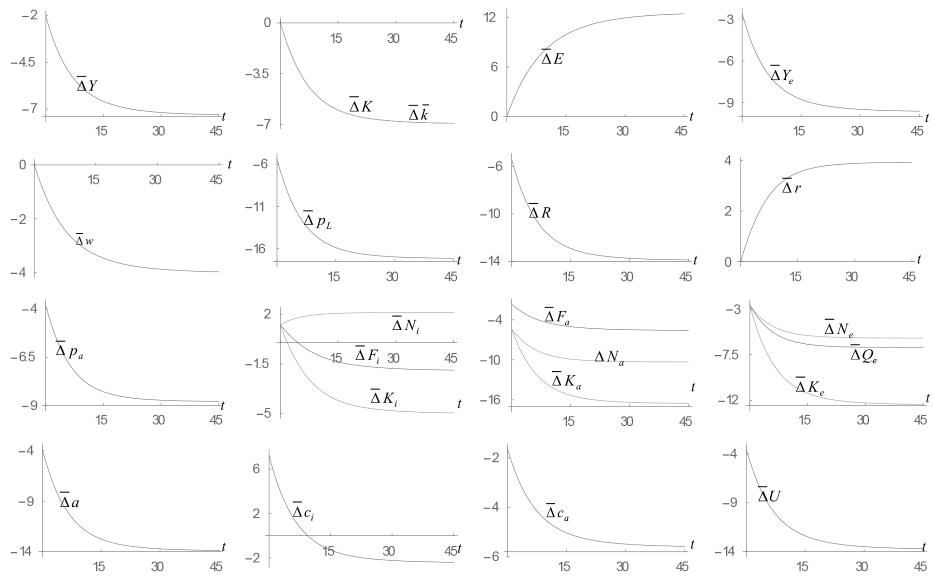

Figure 3. The Propensity to Consume Industrial Goods Is Enhanced

A rise in the propensity to consume agricultural goods

We now study the effects that the propensity to consume agricultural goods is increased as follows: $\mu_0 : 0.06 \Rightarrow 0.07$. The land is reallocated as follows:

$$\overline{\Delta} L_a = 9.22, \ \overline{\Delta} l_h = -6.39, \ \overline{\Delta} \bar{l} = 0.$$

Some of the land is reallocated to the agricultural sector. The effects on the other variables are plotted in Figure 4. As the household spends more of the disposable income on consuming agricultural goods, the output level and capital and labor inputs increase. The total capital stock is reduced and the GDP is augmented. The wage rate is raised initially and reduced in the long term. The rate of interest is enhanced. The land rent and the value of land increase. The household holds more wealth and physical wealth initially and less wealth and physical wealth in the long term. The household's consumption level of agricultural goods rises. The household's consumption level of industrial goods rises initially and falls in the long term. The price of agricultural goods rises. The output level and two input factors of the agricultural sector are augmented. The output level and two input factors of the industrial sector are reduced. The government's income rises in tandem with rising in the output of the environment sector. The environment is enhanced. The household has less wealth and consumes more agricultural goods. The household has higher utility level and consumes more industrial goods initially, and has lower utility level and consumes less industrial goods in the long term.

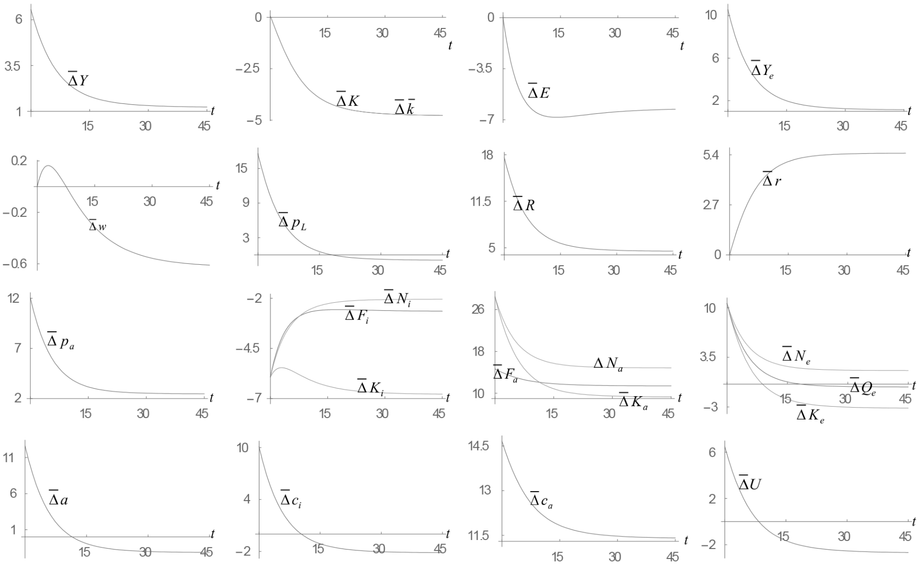

Figure 4. A Rise in the Propensity to Consume Agricultural Goods

A rise in the propensity to save

Like almost any important economic question there are opposing answers to the question if changes in the saving propensity impact on national economic development. In Keynesian economic theory savings tend to reduced national income, while neoclassical growth theory tends to suggest the opposite effect. As only a few growth models with space take account of endogenous land value, economic growth theory has not much to say on how a change in the propensity to save can affect land value and economic structural change in a consistent manner. We now change the propensity to save as follows: $\lambda_0 : 0.7 \Rightarrow 0.71$. There is no impact on land-use allocation. The impact is plotted in Figure 5. The household has more physical wealth and the nation holds more physical capital. The household's wealth falls initially and rises in the long term. This occurs as the land value falls initially and rises in the long term. The GDP also falls initially and rises in the long term. The government's tax income is lowered initially and augmented in the long term. The output level and two inputs of the environment sector are lowered initially and augmented in the long term. The environment deteriorates initially and then improves. The wage rate rises and the rate of interest falls. The land rent and the price of agricultural goods are reduced initially and increased in the long term. The output and two input factors of the industrial sectors are expanded. The output and capital input of the agricultural sector are reduced initially and increased in the long term. The utility level is enhanced.

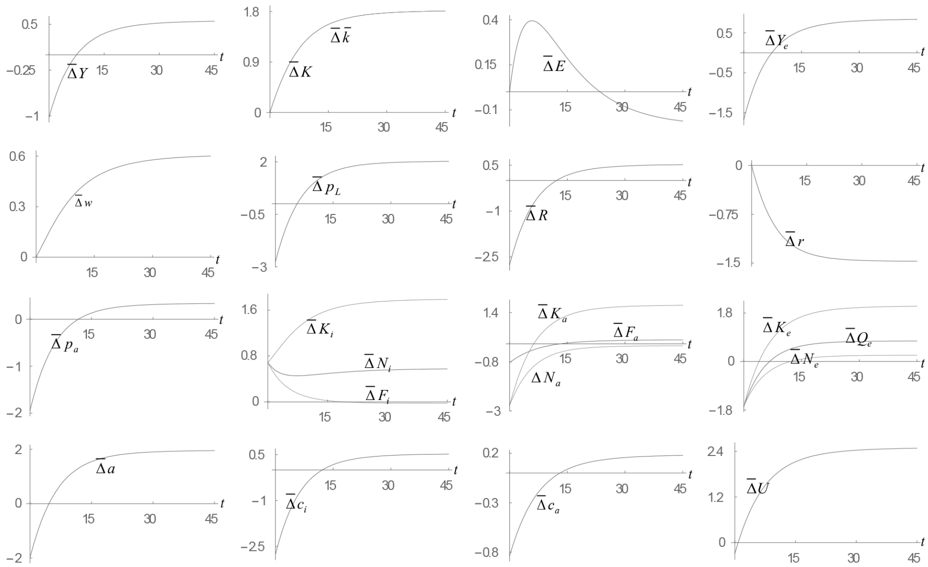

Figure 5. A Rise in the Propensity to Save

An improvement in the environment sector's productivity

We now study the effects of the following improvement of the environment sector's productivity: $A_e : 0.5 \Rightarrow 0.6$. There is no impact on land-use allocation. The impact is plotted in Figure 6. The output of the environment sector is enhanced. The environment improves. The output levels of the industrial and agricultural sectors increase. The utility level is also enhanced. The labor distribution is slightly affected. The household has more physical wealth and the nation holds more physical capital. The household's total wealth rises. The GDP rises. The government's tax income is increased. The capital input of the environment sector is augmented. The wage rate and the rate of interest rise. The land rent is enhanced. The price of agricultural goods is reduced initially and increased in the long term. The land value increases. The household consumes more goods.

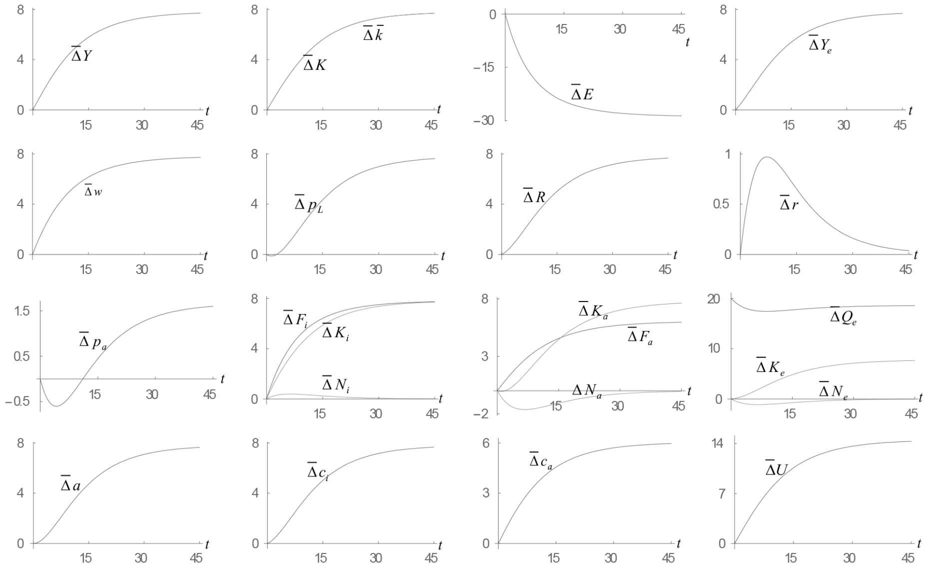

Figure 6. An Improvement in the Environment Sector's Productivity

A rise in the tax rate on consumption of industrial goods

We now study the effects that the tax rate on consumption of industrial goods is increased as follows: $\tau_i : 0.01 \Rightarrow 0.015$. The land use pattern is not affected. The effects on the other variables are plotted in Figure 7. The consumption of industrial goods is reduced and the consumption of agricultural goods is slightly augmented. The total capital stock and GDP are slightly augmented. The wage rate, the rate of interest, the land rent and the value of land are slightly increased. The household holds more wealth and physical wealth. The price of agricultural goods is slightly affected. The output level and capital inputs of the agricultural and industrial sectors are augmented. The environment is enhanced.

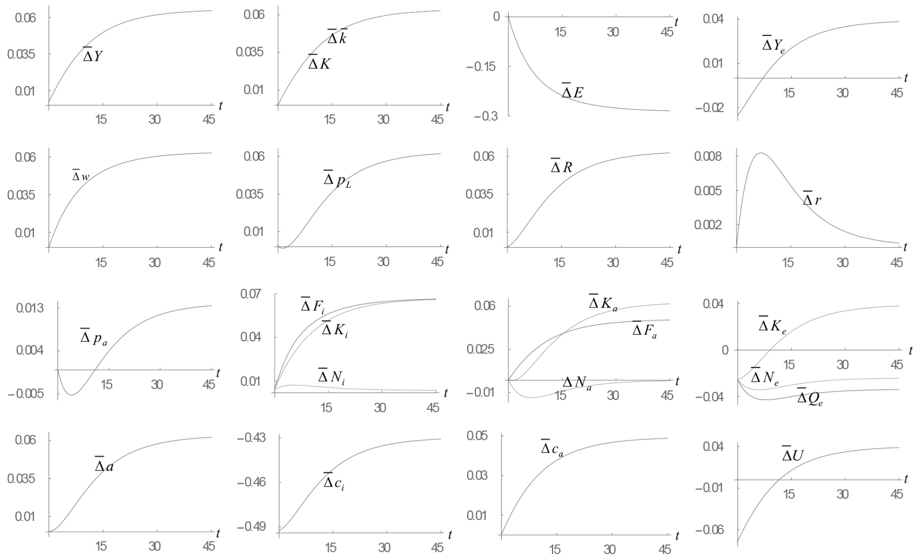

Figure 7.A Rise in the Tax Rate on Consumption of Industrial Goods

A rise in the tax rate in the industrial sector

We now study the effects when the tax rate on consumption of industrial goods is increased as follows: $\tau_i : 0.01 \Rightarrow 0.015$. The land use pattern is not affected. The effects on the other variables are plotted in Figure 8. Comparing Figures 7 and 8, we see that the long-term effects are similar.

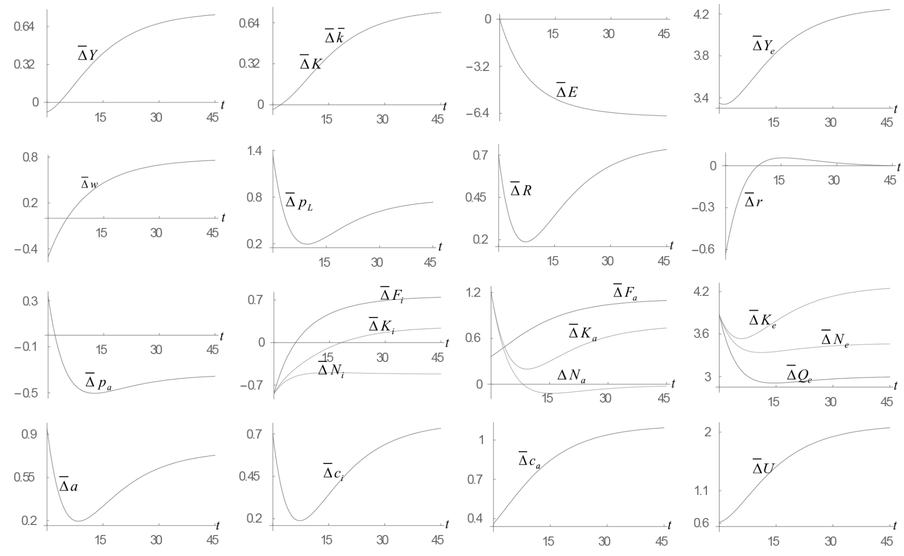

Figure 8. A Rise in the Tax Rate in the Industrial Sector's Output

An increase in the population

We now study the effects of the following population increase: $N: 20 \Rightarrow 21$. The impact on land-use allocation is as follows:

$$\bar{\Delta} L_a = 0,\ \bar{\Delta} l_h = -4.76,\ \bar{\Delta} \bar{l} = -4.76.$$

The land use for agriculture is not affected. The lot size and the per household land are reduced. The effects on the other variables are plotted in Figure 9. The GDP and national physical capital are augmented. The environment deteriorates. The government receives more income initially and less in the long term. The capital input rises initially and falls in the long term. The output level and labor input of the environment sector are increased. Both wage rate and rate of interest are lowered. The output levels and capital and labor inputs of the industrial and agricultural sectors are augmented. The utility level is lowered. The household has less physical wealth and total wealth. The land rent is enhanced. The price of agricultural goods and the land value are increased. The household consumes less goods.

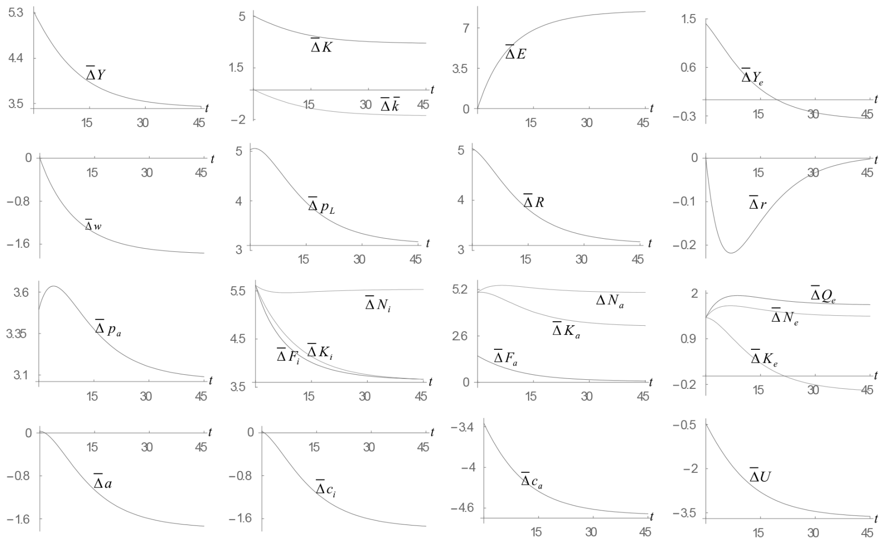

Figure 9. An Increase in the Population

A rise in the tax rate in the land rent income

We now study the effects that the tax rate on land ownership is increased as follows: $\tau_R : 0.01 \Rightarrow 0.015$. The land-use pattern is not affected. The effects on the other variables are plotted in Figure 10. As the household has to pay more tax on owning housing, the land value and GDP fall initially. The household increases physical wealth.

The national physical capital is augmented. In the long term the land value and the GDP are augmented. The wage rate rises and the rate of interest falls. The price of agricultural goods falls. The output level and two input factors of the agricultural sector are reduced initially and augmented. The output level and labor input of the industrial sector are increased and the sector's capital input is reduced. The government's income rises in tandem with rises in the output of the environment sector. The environment is enhanced. The household has less wealth initially and more in the long term. The household has lower consumption levels of agricultural and industrial goods and lower utility initially and these variables are enhanced in the long term.

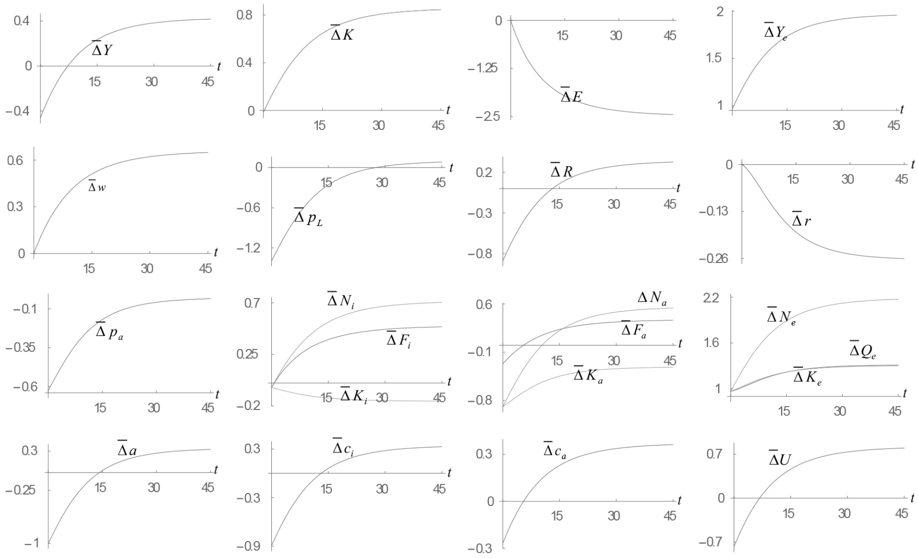

Figure 10. A Rise in the Tax Rate on the Land Rent Income

A rise in the tax rate on consuming housing

We now study the effects of the following change in that the tax rate on consuming housing: $\tau_R : 0.01 \Rightarrow 0.015$. The land is reallocated as follows:

$$\overline{\Delta} L_a = 0.29, \ \overline{\Delta} l_h = -0.2, \ \overline{\Delta} \bar{l} = 0.$$

Some of the land is reallocated to the agricultural sector. The effects on the other variables are plotted in Figure 11. As the household has to pay more in consuming the same amount of housing, the lot size and land rent falls. The government receives less tax income. The environment produces less and uses less capital and labor inputs. The environment is deteriorated. The household owns more physical capital and the total capital stock is augmented. The GDP is reduced. The wage rate and the rate of interest are reduced. The land value is reduced. The industrial sector increases its output level and capital and labor inputs. The agricultural sector increases its output level and reduces its capital and labor inputs in association with rising in the land input. The price of agricultural goods falls and the household's consumption level of agricultural goods rises. The household's consumption level of industrial goods falls. The household has a lower utility level.

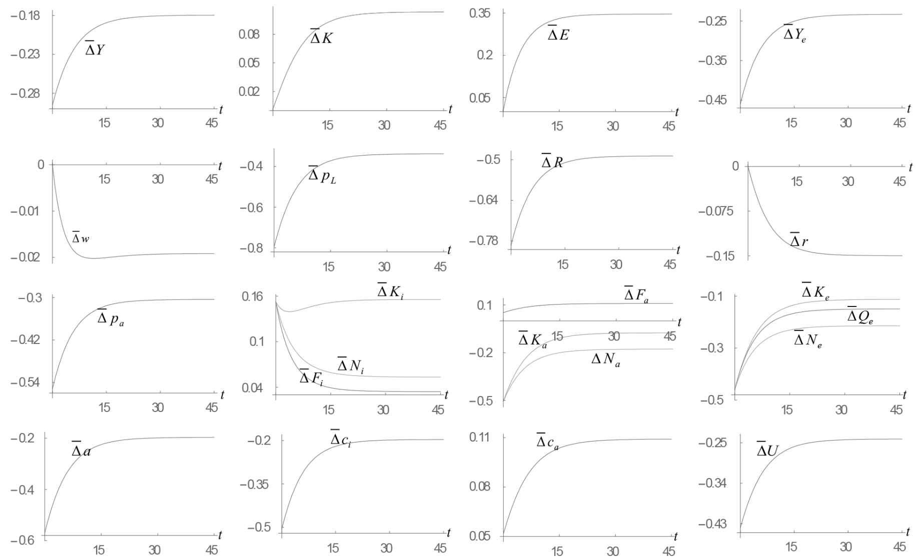

Figure 11. A Rise in the Tax Rate on Consuming Housing

5. Concluding remarks

This study examined dynamic interactions among land, capital, economic structure, and environment. The main framework is neoclassical and the household's decision is based on an alternative approach by Zhang. We integrated some ideas in the neoclassical growth theory, environmental economics and land economics in a compact framework. The national economy consists of three – industrial, agricultural, and environment – sectors. The production side is the same as in the neoclassical growth theory. We simulated the motion of the model and examined the effects of changes in the propensity to consume housing, the propensity to consume industrial goods, the propensity to consume agricultural goods, the propensity to save, the environment sector's productivity, the tax rate on consumption of industrial goods, the tax rate on the industrial sector, the population, the tax rate on owning land, the tax rate on consuming housing, and the tax rate on consuming housing. The comparative dynamic analyses provide some important insights. For instance our simulation results in relations between economic growth and land value being similar to the phenomenon that is described by Liu *et al*. (2011: 1), "The recent financial crisis caused by a collapse of the housing market propelled the U.S. economy into the Great Recession. A notable development during the crisis period was a slump in business investment in tandem with a sharp decline in land prices." The similarity is concluded in the sense that a rising period in the GDP is in tandem with a rising period in the land value. Indeed, our conclusion is obtained under the strict conditions without taking account of many possible important determinants of land prices. It should be noted that many limitations of this model become apparent in the light of the sophistication of the literature of growth theory and land economics. The model can be extended and generalized in different directions. For instance, it is important to study the economic dynamics when utility and production functions are taken on other functional forms.

Appendix: Proving the lemma

The appendix shows that the dynamics can be expressed by two differential equations. From (2), (4), and (22) we obtain

$$z \equiv \frac{r + \delta_k}{w} = \frac{\tilde{\alpha}_i N_i}{K_i} = \frac{\tilde{\alpha}_a N_a}{K_a} = \frac{\tilde{\alpha}_e N_e}{K_e},$$

where we omit the time index and $\tilde{\alpha}_j \equiv \alpha_j / \beta_j$, $j = i, a, e$. By (1) and (2), we have

$$r + \delta_k = \frac{\bar{\tau}_i \alpha_i A_i}{\tilde{\alpha}_i^{\beta_i}} \Gamma_i z^{\beta_i}, \quad w = \frac{\bar{\tau}_i \tilde{\alpha}_i^{\alpha_i} \beta_i A_i \Gamma_i}{z^{\alpha_i}},$$

where we also use (A1). We express w and r as functions of z and E.

From (12) and (10), we get

$$\mu \hat{y} N = p_a F_a.$$

From (4), we have

$$r + \delta_k = \frac{\bar{\tau}_a \alpha_a p_a F_a}{K_a}.$$

From (A4) and (3) we get

$$p_a \left(\frac{L_a}{K_a} \right)^{\varsigma} = \frac{\tilde{\alpha}_a^{\beta_a} (r + \delta_k)}{\bar{\tau}_a \alpha_a A_a \Gamma_a z^{\beta_a}},$$

where we use (A1). From (A3) and (A4), we get

$$\mu \hat{y} N = \left(\frac{r + \delta_k}{\bar{\tau}_a \alpha_a} \right) K_a.$$

By (4) and (A3), we have

$$R = \frac{\bar{\tau}_a \varsigma \mu \hat{y} N}{L_a}.$$

From $R l_h = \eta \hat{y}$ in (10) and (A7), we have

$$\bar{\tau}_a \varsigma \mu N l_h = \eta L_a.$$

From (17) and (A8), we calculate the land distribution as follows:

$$L_a = \frac{\bar{\tau}_a \varsigma \mu L}{\eta + \bar{\tau}_a \varsigma \mu}, \; l_h = \frac{\eta L}{(\eta + \bar{\tau}_a \varsigma \mu) N}.$$

The land distribution is invariant over time.

From the definition of $\hat{y}$, we have

$$\hat{y} = (1 + (1 - \tau_k) r)\bar{k} + (1 - \tau_w) w + (1 - \tau_R)\frac{RL}{N} + p_L \frac{L}{N},$$

where we also use (13). Insert (5) in (A10)

$$\hat{y} = (1 + (1 - \tau_k) r)\bar{k} + (1 - \tau_w) w + \left[1 + \frac{1}{(1 - \tau_k) r}\right]\frac{(1 - \tau_R) L R}{N}.$$

From $R l_h = \eta \hat{y}$ in (10) and (A11) we calculate

$$R = \omega_1 \bar{k} + \omega_2,$$

where

$$\omega_0(z, E) \equiv \left\{\frac{l_h}{\eta} - \left[1 + \frac{1}{(1 - \tau_k) r}\right]\frac{(1 - \tau_R) L}{N}\right\}^{-1},$$
$$\omega_1(z, E) \equiv (1 + (1 - \tau_k) r)\omega_0, \; \omega_2(z, E) \equiv (1 - \tau_w) w \omega_0.$$

From (A11) and (A12) we have

$$\hat{y} = \tilde{\omega}_1 \bar{k} + \tilde{\omega}_2,$$

where

$$\tilde{\omega}_1(z, E) \equiv (1 + (1 - \tau_k) r) + \left[1 + \frac{1}{(1 - \tau_k) r}\right]\frac{(1 - \tau_R) L \omega_1}{N},$$
$$\tilde{\omega}_2(z, E) \equiv (1 - \tau_w) w + \left[1 + \frac{1}{(1 - \tau_k) r}\right]\frac{(1 - \tau_R) L \omega_2}{N}.$$

Insert (A1) in $N_i + N_a + N_e = N$

$$\frac{K_i}{\tilde{\alpha}_i} + \frac{K_a}{\tilde{\alpha}_a} + \frac{K_e}{\tilde{\alpha}_e} = \frac{N}{z}.$$

From (14) and (15) we have

$$K_i + K_a + K_e = N\bar{k}.$$

From (A6) and (A13) we calculate

$$K_a = \hat{\omega}_1 \bar{k} + \hat{\omega}_2,$$

where

$$\hat{\omega}_1(z, E) \equiv \tilde{\omega}_1 \mu N\left(\frac{\alpha_a \bar{\tau}_a}{r + \delta_k}\right), \quad \hat{\omega}_2(z, E) \equiv \tilde{\omega}_2 \mu N\left(\frac{\alpha_a \bar{\tau}_a}{r + \delta_k}\right).$$

Insert (A16) in, respectively, (A14) and (A15)

$$\frac{K_i}{\tilde{\alpha}_i} + \frac{K_e}{\tilde{\alpha}_e} = b_1(z, E) \equiv \frac{N}{z} - \frac{\hat{\omega}_2}{\tilde{\alpha}_a} - \frac{\hat{\omega}_1 \bar{k}}{\tilde{\alpha}_a},$$
$$K_i + K_e = b_2(z, E) \equiv N\bar{k} - \hat{\omega}_1 \bar{k} - \hat{\omega}_2.$$

Calculate (A17)

$$K_i = \alpha_0 b_1 - \frac{\alpha_0 b_2}{\tilde{\alpha}_e}, \quad K_e = \frac{\alpha_0 b_2}{\tilde{\alpha}_i} - \alpha_0 b_1,$$

where

$$\alpha_0 \equiv \left(\frac{1}{\tilde{\alpha}_i} - \frac{1}{\tilde{\alpha}_e}\right)^{-1}.$$

Insert the definitions of b_j in (A18)

$$K_i = m_i \bar{k} - \bar{m}_i, \quad K_e = m_e \bar{k} - \bar{m}_e,$$

where

$$m_i(z, E) \equiv -\frac{\alpha_0 \hat{\omega}_1}{\tilde{\alpha}_a} - \frac{\alpha_0}{\tilde{\alpha}_e} N + \frac{\alpha_0}{\tilde{\alpha}_e} \hat{\omega}_1, \quad \bar{m}_i(z, E) \equiv \frac{\alpha_0 \hat{\omega}_2}{\tilde{\alpha}_a} - \frac{\alpha_0 N}{z} - \frac{\alpha_0}{\tilde{\alpha}_e} \hat{\omega}_2,$$
$$m_e(z, E) \equiv \frac{\alpha_0 N}{\tilde{\alpha}_i} - \frac{\alpha_0 \hat{\omega}_1}{\tilde{\alpha}_i} + \frac{\alpha_0 \hat{\omega}_1}{\tilde{\alpha}_a}, \quad \bar{m}_e(z, E) \equiv \frac{\alpha_0 \hat{\omega}_2}{\tilde{\alpha}_i} + \frac{\alpha_0 N}{z} - \frac{\alpha_0 \hat{\omega}_2}{\tilde{\alpha}_a}.$$

By (A8), we solve the capital distribution as functions of z, E and $\bar{k}$. By (A1), we solve the labor distribution as functions of z, E and $\bar{k}$ as follows

$$N_i = \frac{z K_i}{\widetilde{\alpha}_i}, \;\; N_a = \frac{z K_a}{\widetilde{\alpha}_a}, \;\; N_e = \frac{z K_e}{\widetilde{\alpha}_e}.$$

From (5)

$$p_L = \frac{(1 - \tau_R) R}{(1 - \tau_k) r}.$$

Insert (2) and (A18) in (20)

$$Y_e = \left(\frac{r + \delta_k}{\alpha_i \, \overline{\tau}_i}\right) \tau_i \, K_i + \left(\frac{r + \delta_k}{\alpha_a \, \overline{\tau}_a}\right) \tau_a \, K_a + I_c \, .$$

Substituting (A16) and (A19) into (A22) yields

$$Y_e = I_k \, \bar{k} - I_0 + I_c \, ,$$

where

$$I_k(z, E) \equiv \left(\frac{r + \delta_k}{\alpha_i \, \overline{\tau}_i}\right) \tau_i \, m_i + \left(\frac{r + \delta_k}{\alpha_a \, \overline{\tau}_a}\right) \tau_a \, \hat{\omega}_1 , \;\; I_0(z, E) \equiv \left(\frac{r + \delta_k}{\alpha_i \, \overline{\tau}_i}\right) \tau_i \, \overline{m}_i - \left(\frac{r + \delta_k}{\alpha_a \, \overline{\tau}_a}\right) \tau_a \, \hat{\omega}_2 \, .$$

Insert (10) in the definition of I_c

$$I_c = \tau_0 \, \hat{y} + \tau_k \, r \, N_0 \, \bar{k} + \tau_w \, N_0 \, w,$$

where

$$\tau_0 \equiv \left(\tau_c \, \xi + \tau_a \, \mu + \tau_h \, \eta + \frac{\eta \, \tau_R \, \bar{l}}{l_h}\right) N_0 \, .$$

Insert (A13) in (A24)

$$I_c = (\tau_0 \, \widetilde{\omega}_1 + \tau_0 \, \tau_k \, r \, N_0) \bar{k} + \widetilde{\omega}_2 + \tau_w \, N_0 \, w.$$

Insert (A25) in (A23)

$$Y_e = (I_k + \tau_0 \, \widetilde{\omega}_1 + \tau_0 \, \tau_k \, r \, N_0) \bar{k} - I_0 + \widetilde{\omega}_2 + \tau_w \, N_0 \, w.$$

From (22) and (A26) we calculate

$$\left(\frac{r + \delta_k}{\alpha_e}\right) K_e = (I_k + \tau_0 \, \widetilde{\omega}_1 + \tau_0 \, \tau_k \, r \, N_0) \bar{k} - I_0 + \widetilde{\omega}_2 + \tau_w \, N_0 \, w.$$

Insert (A19) in (A27)

$$\bar{k}(z, E) = \left[\left(\frac{r+\delta_k}{\alpha_e}\right)\overline{m}_e - I_0 + \widetilde{\omega}_2 + \tau_w N_0 w\right]\left[\left(\frac{r+\delta_k}{\alpha_e}\right)m_e - \left(I_k + \tau_0 \widetilde{\omega}_1 + \tau_0 \tau_k r N_0\right)\right]^{-1}.$$

From (6) and (A28) we have

$$a = \phi(z, E) \equiv \bar{k} + \frac{(1-\tau_R)\bar{l} R}{(1-\tau_k)r}.$$

It is straightforward to check that all the variables can be expressed as functions of z and E at any point in time as follows: r and w by (A2) $\rightarrow$ $\bar{k}$ by (A28) $\rightarrow$ K_a by (A6) $\rightarrow$ K_i and K_e by (A19) $\rightarrow$ N_i, N_e, and N_a by (A1) $\rightarrow$ $\hat{y}$ by (A13) $\rightarrow$ $\bar{l}$ by (13) $\rightarrow$ R by (A12) $\rightarrow$ p_L by (A21) $\rightarrow$ a by (A23) $\rightarrow$ L_a and l_h by (A9) $\rightarrow$ p_a by (A5) $\rightarrow$ c_i, c_a, and s by (10) $\rightarrow$ F_i by (1) $\rightarrow$ F_a by (3) $\rightarrow$ Y_e by (22) $\rightarrow$ Q_e by (19). From this procedure, (18) and (11), we have

$$\dot{a} = \Lambda_0(z, E) \equiv s - a,$$

(A30)

$$\dot{E} = \Omega(z, E) \equiv \theta_i F_i + \theta_a F_a + \widetilde{\theta}_i C_i + \widetilde{\theta}_a C_a - Q_e - \theta_0 E.$$

Taking derivatives of (A29) with respect to t yields

$$\dot{a} = \frac{\partial \phi}{\partial z}\dot{z} + \Omega \frac{\partial \phi}{\partial E},$$

in which we use (A31). Equal (A30) and (A32)

$$\dot{z} = \Lambda(z, E) \equiv \left(\Lambda_0 - \Omega \frac{\partial \phi}{\partial E}\right)\left(\frac{\partial \phi}{\partial z}\right)^{-1}.$$

From (A31) and (A33), we determine the motion of z and E. We thus proved the lemma.

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